

Over the HH-model

Different currents	Function
slow K ⁺	firing frequency adaptation
perzistent (non-inactivating)	Na ⁺ burst
Ca ²⁺	burst
Ca ²⁺ -(and V-)-dependent K ⁺ burst, adaptation	
H hyperpolarization activated	pacemaker

Voltage dependent currents

$$I_X = g_X(t)(E_X - V(t)), \quad g_X(t) = \bar{g}_X \prod_i p_X^{(i)}(t), \quad \frac{dp_X^{(i)}}{dt} = \frac{p_{X\infty}^{(i)}(V(t)) - p(t)}{\tau_{p_X^{(i)}}(V(t))}$$

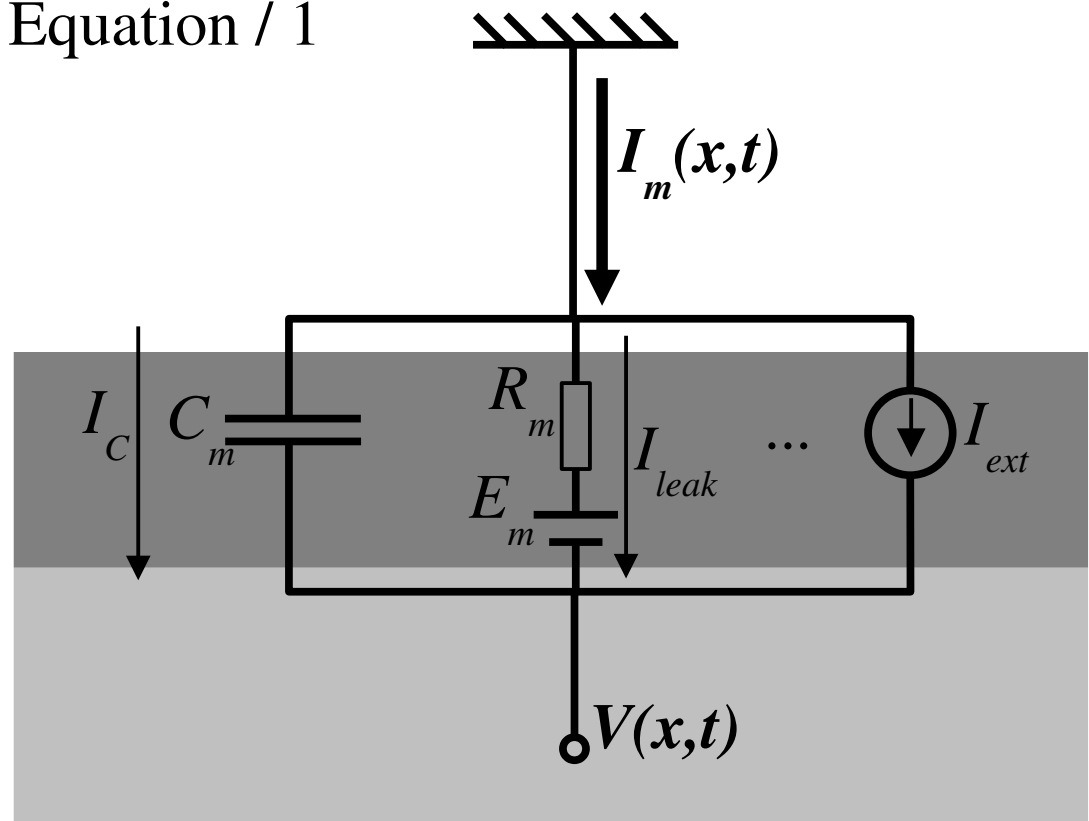
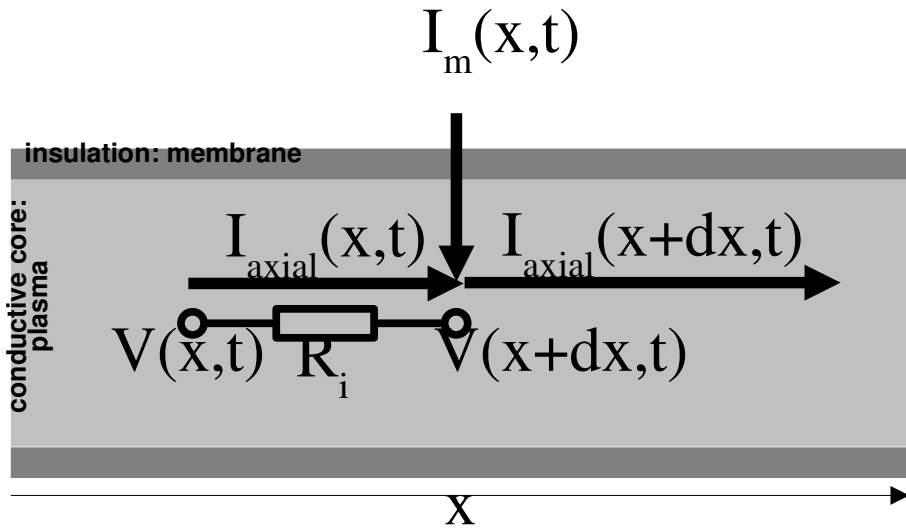
Ca²⁺ concentration

$$\frac{d[Ca]_i}{dt} = \beta I_{Ca}(t) - \frac{[Ca]_i(t)}{\tau_{Ca}}$$

Ca²⁺-dependent gate

$$\frac{dp_X^{(i)}}{dt} = \frac{p_{X\infty}^{(i)}([Ca]_i(t)) - p(t)}{\tau_{p_X^{(i)}}([Ca]_i(t))}$$

The Cable Equation / 1



$$I_{axial}(x,t) = -\frac{1}{R_i} \frac{\partial V}{\partial x}$$

$$\frac{\partial I_{axial}}{\partial x} = I_m(x,t)$$

$$I_m(x,t) = I_c(x,t) + I_{leak}(x,t) + \dots = -C_m \frac{\partial V}{\partial t} - \frac{V(x,t)}{R_m}$$

simplification: no voltage-dependent currents!

x [cm], t [msec]
 V [mV], I_{axial} [μ A], I_m [μ A/cm]
 R_i [k Ω /cm], R_m [k Ω cm], C_m [μ F/cm]

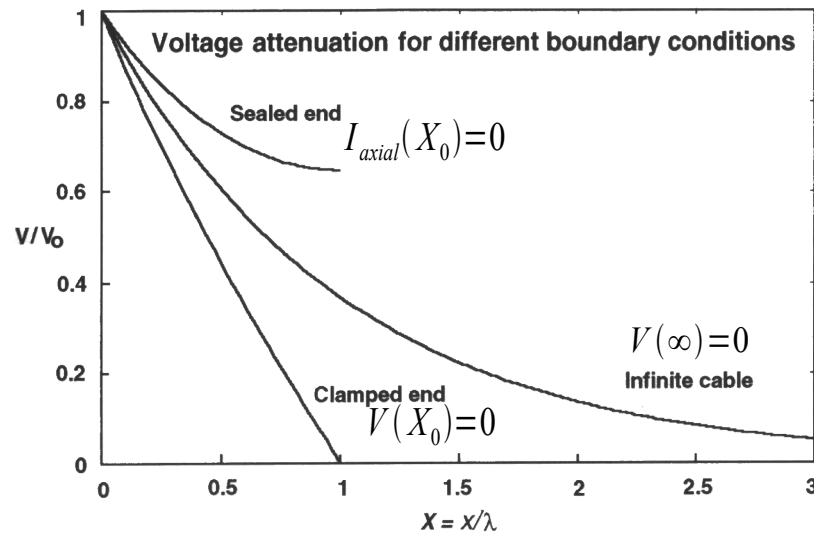
$$\lambda = \sqrt{R_m / R_i} \text{ [cm]}$$

$$\tau = R_m C_m \text{ [msec]}$$

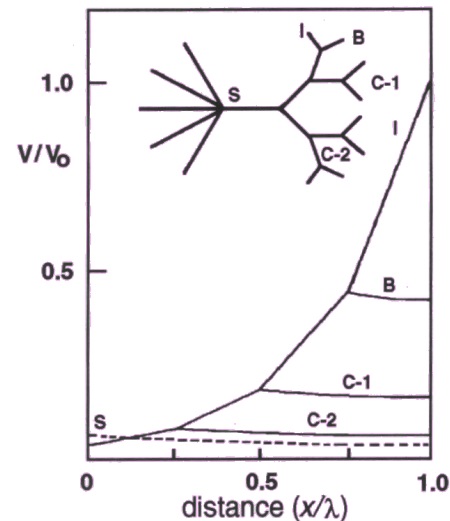
$$\frac{1}{R_i} \frac{\partial^2 V}{\partial x^2} - C_m \frac{\partial V}{\partial t} - \frac{V(x,t)}{R_m} = 0$$

$$\lambda^2 \frac{\partial^2 V}{\partial x^2} - \tau \frac{\partial V}{\partial t} - V(x,t) = 0$$

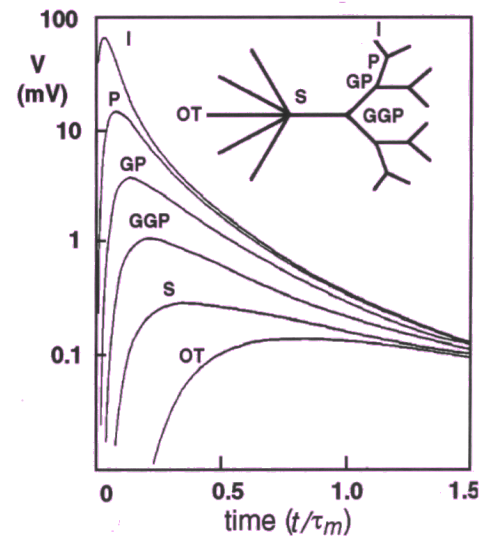
The Cable Equation / 2



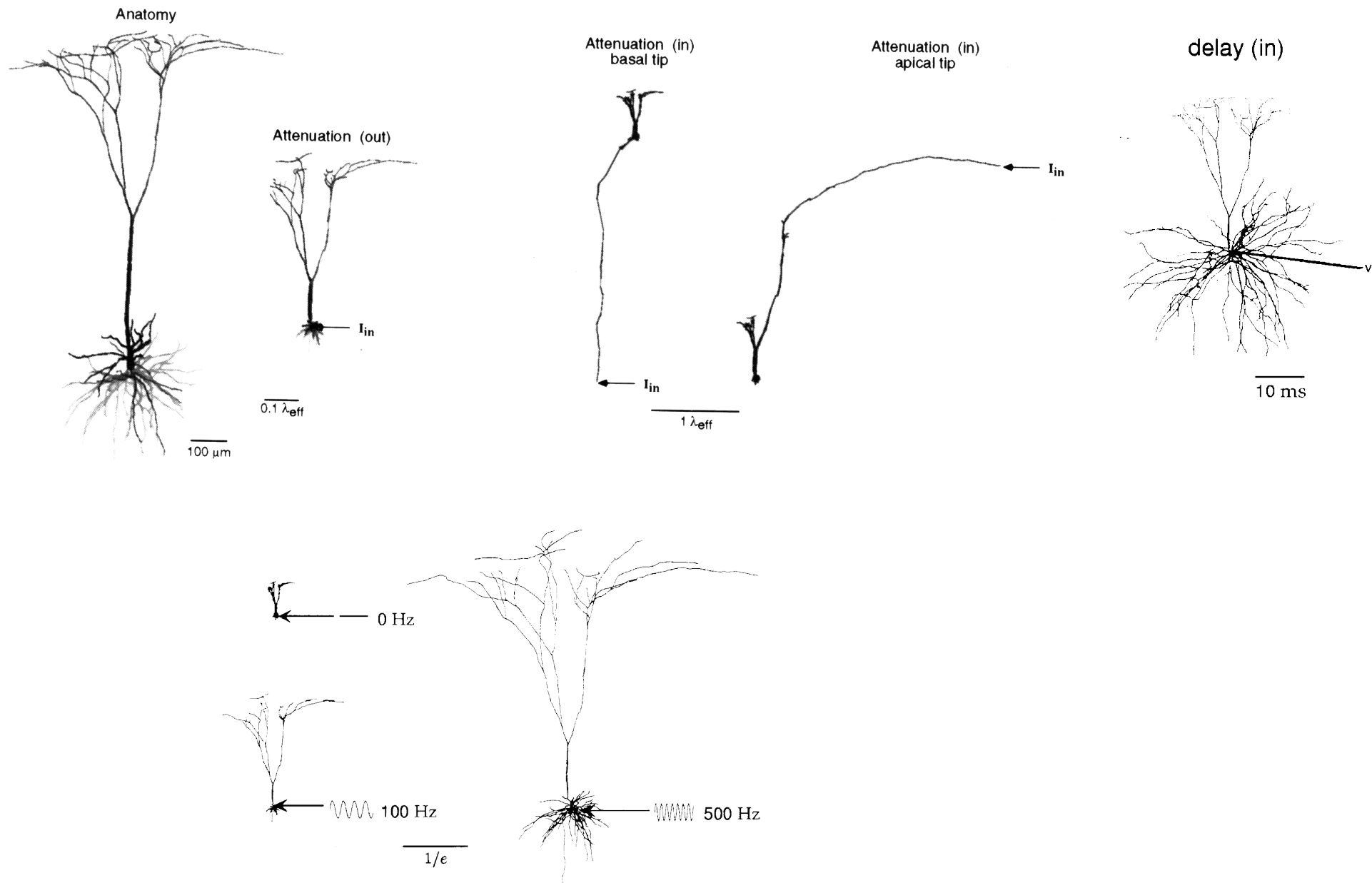
Constant current injection:
steady-state **spatial**
voltage spread



Transient current injection:
temporal development
of voltage spread



The Cable Equation / 3



Multicompartmental modeling

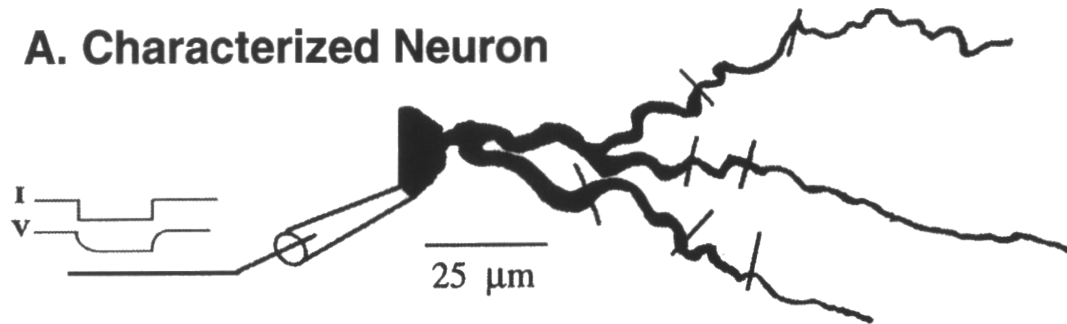
$$\frac{1}{R_i} \frac{\partial^2 V}{\partial x^2} - C_m \frac{\partial V}{\partial t} - \frac{V(x, t)}{R_m} = 0$$



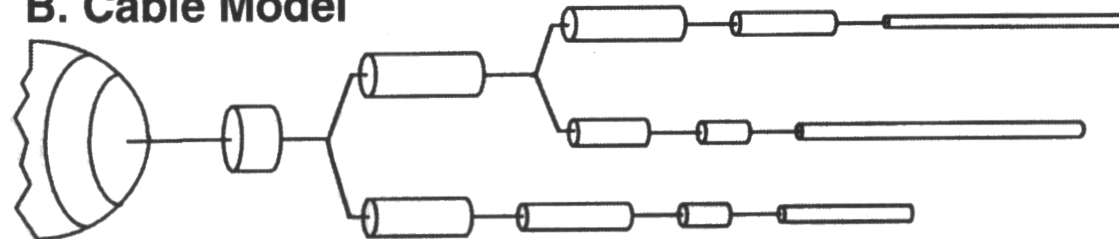
$$C_k \frac{dV_k}{dt} = \underbrace{I_k(t)} + \frac{V_{k-1}(t) - V_k(t)}{R_k} + \frac{V_{k+1}(t) - V_k(t)}{R_{k+1}}$$

all sorts of ionic currents (HH, etc)

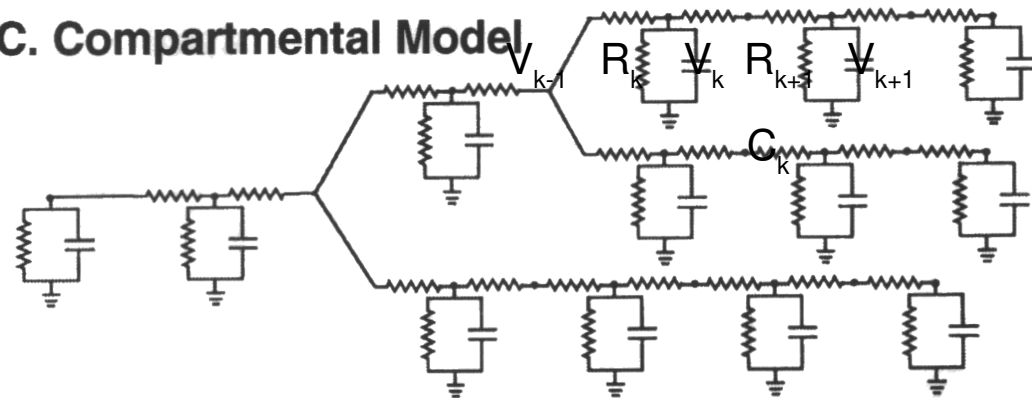
A. Characterized Neuron



B. Cable Model



C. Compartmental Model



Detailed cell models: Why?

I. Reproducing different phenomena (how does it works?)

Traub & Miles (1991, 1994) hippocampal pyramidal cell model

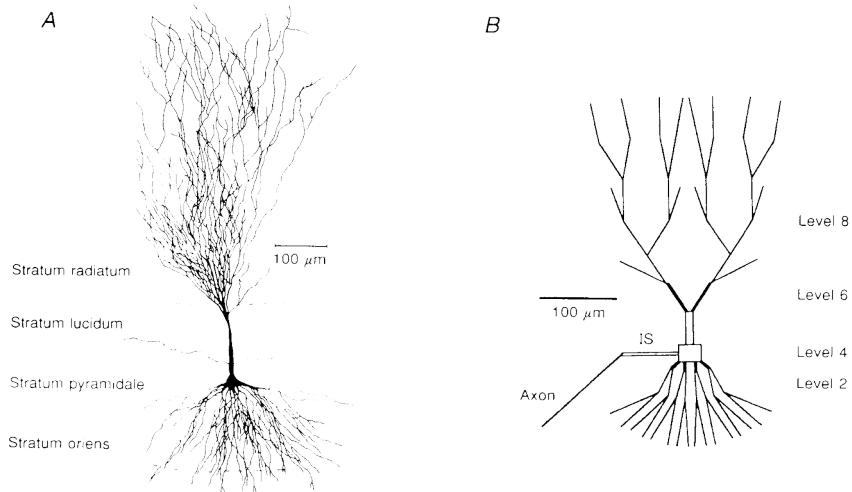
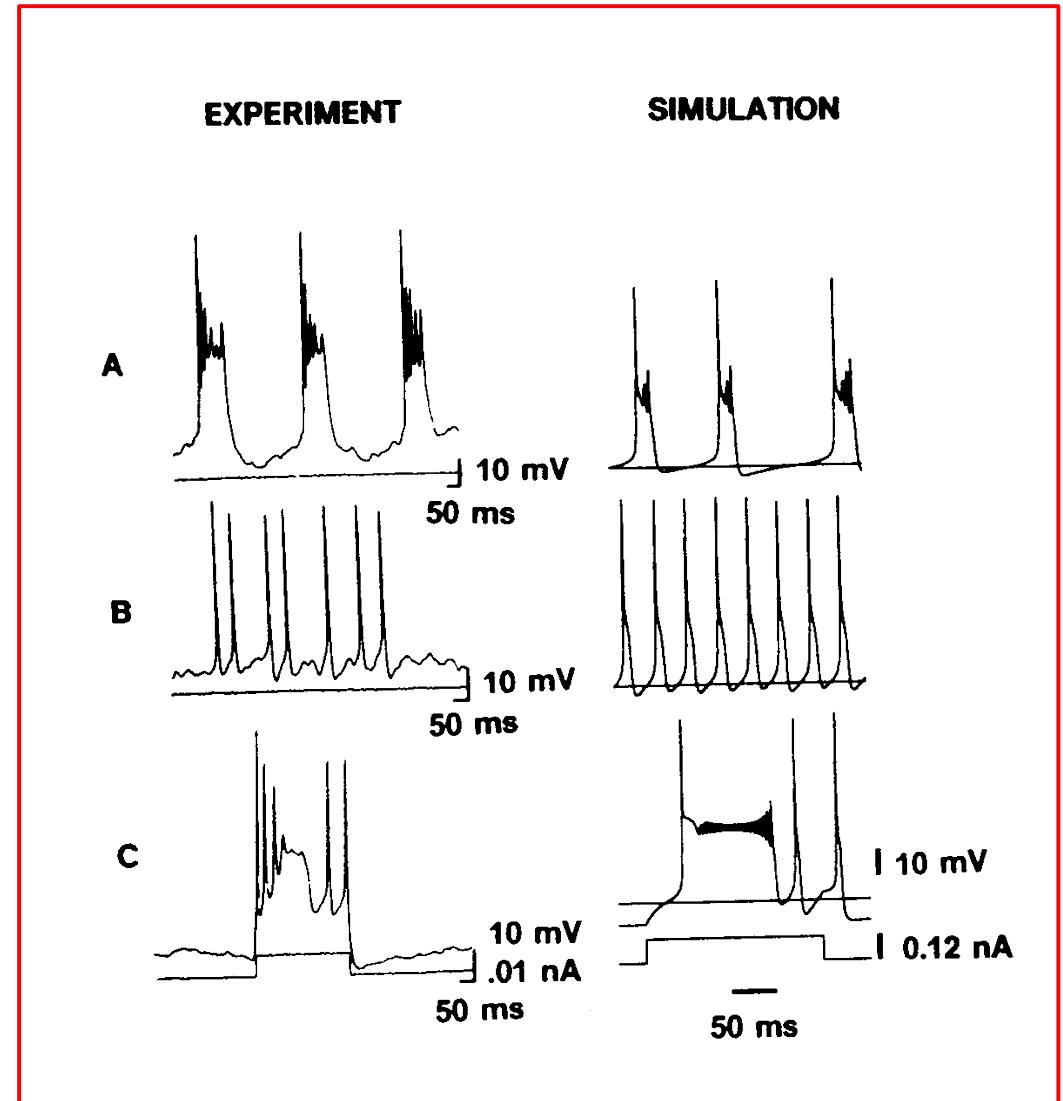


Table 2. Active conductance densities (mS cm^{-2})

Level	Na^+	Ca^{2+}	K(DR)	K(AHP)	K(C)	K(A)
1	—	1.0	—	0.8	4.0	—
2	—	1.0	—	0.8	4.0	0.5
3	1.0	1.0	15	0.8	8.0	0.5
4	100	1.0	135	0.8	20	0.5
5	3.0	1.0	20	0.8	8.0	0.5
6	3.0	1.0	20	0.8	8.0	0.5
7	—	2.0	—	0.8	4.0	—
8	—	3.0	—	0.8	12	—
9	—	3.0	—	0.8	12	—
10	—	1.0	—	0.8	4.0	—
11	—	1.0	—	0.8	4.0	—
IS	500	—	250	—	—	—
Axon	500	—	250	—	—	—

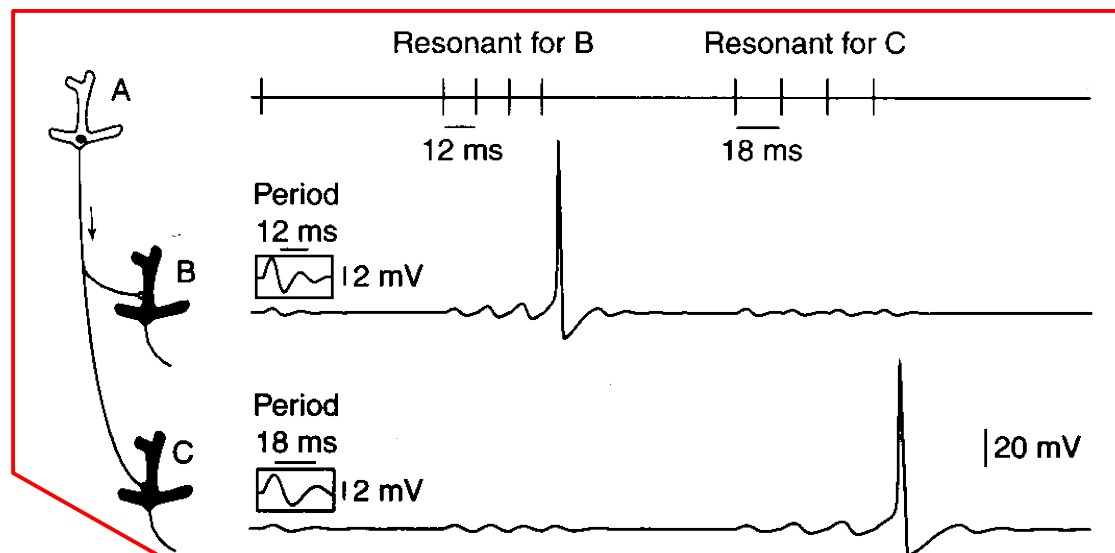
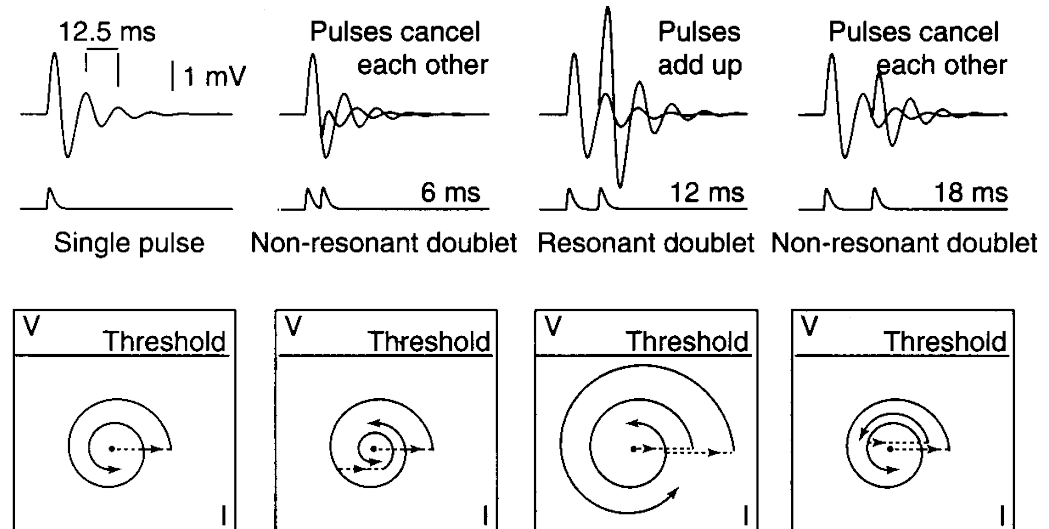
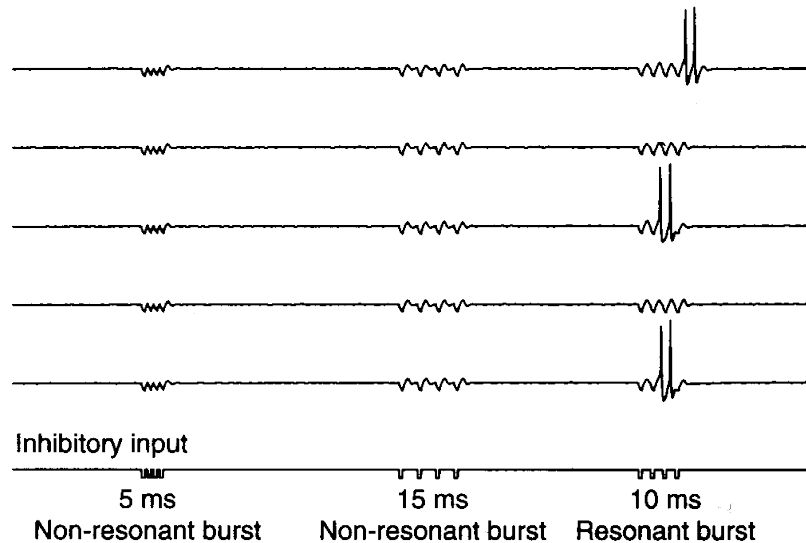


Detailed cell models: Why?

II. Revealing computational functions (What it is good for?) / 1

What is a burst good for?

1. Common sense (Lisman): more robust transmission
2. Izhikevich: selective communication with resonance

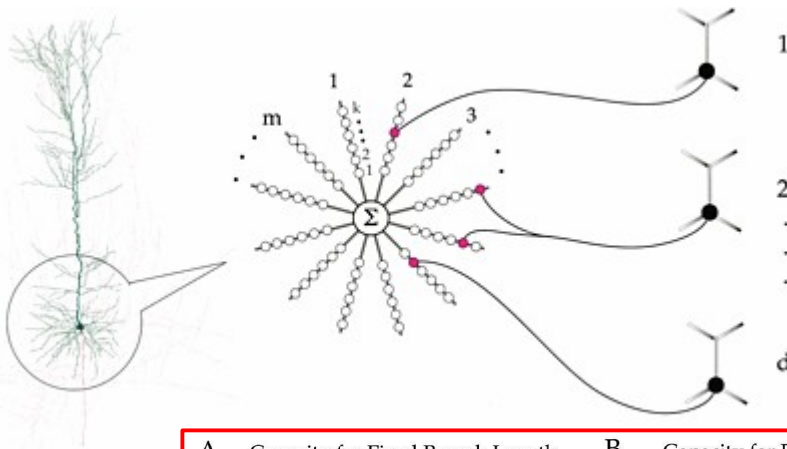


Detailed cell models: why?

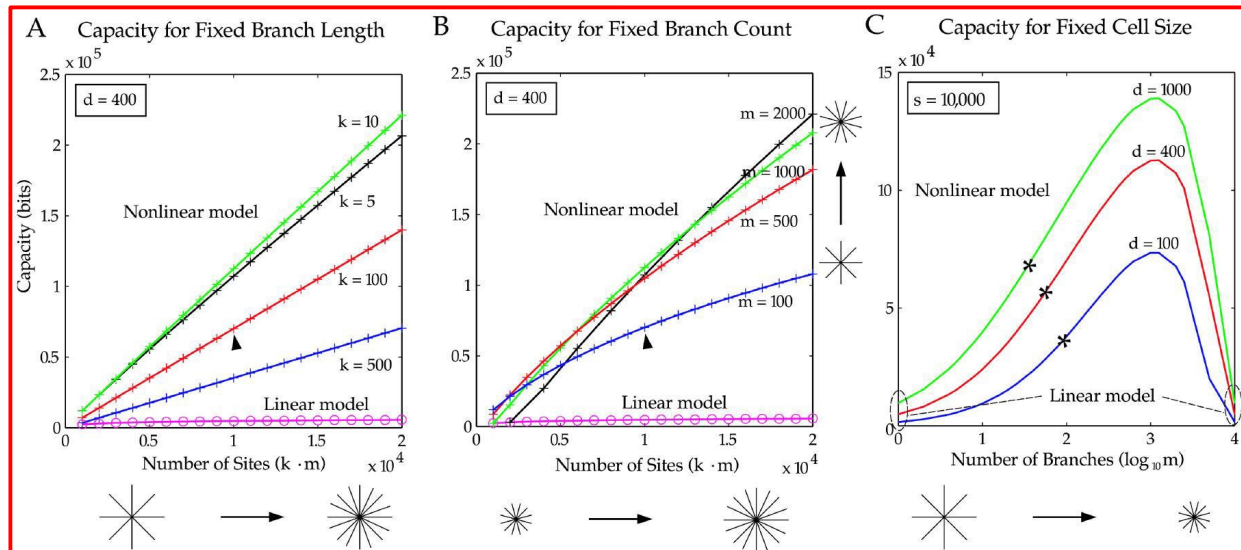
II. Revealing computational functions (What it is good for?) / 2

What is the role of the dendrites?

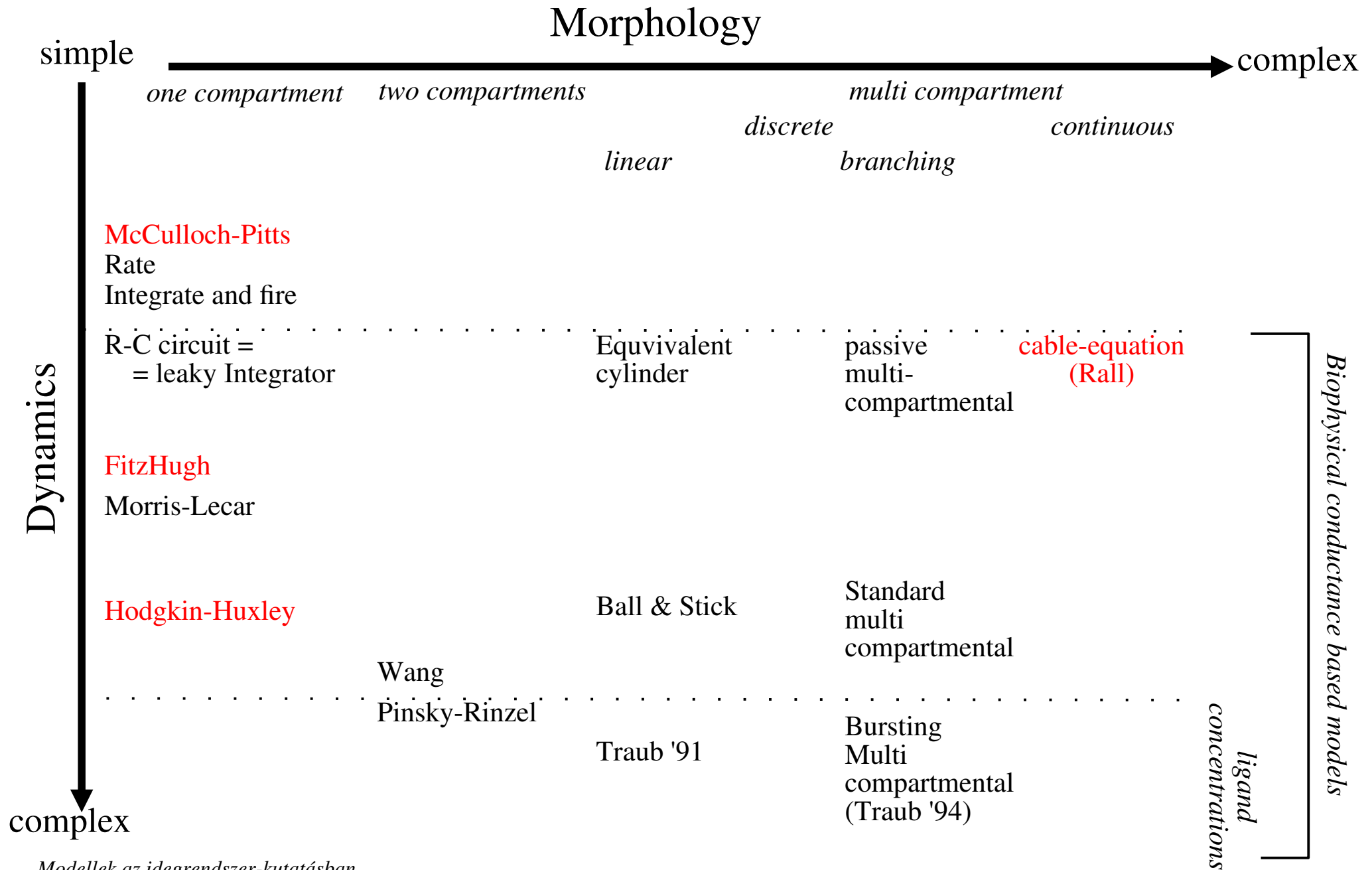
1. Common sense (Cook&Johston): amplification of distal synaptic effects
2. Mel: increasing storage capacity



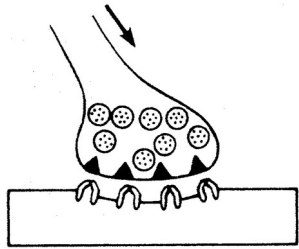
	Linear Cell	Nonlinear Cell
$\circ d_1$ $\bullet d_2$ $\bullet d_3$	$a_L(x)$	$a_N(x)$
$d, m, k = 3$		
Wiring Configurations ①	$4x_1 + 3x_2 + 2x_3$	$b(2x_1 + x_2) +$ $b(2x_1 + x_2) +$ $b(x_2 + 2x_3)$
②	$4x_1 + 3x_2 + 2x_3$	$b(2x_1 + x_3) +$ $b(x_1 + 2x_2) +$ $b(x_1 + x_2 + x_3)$
Total number of distinct i/o functions	110	220



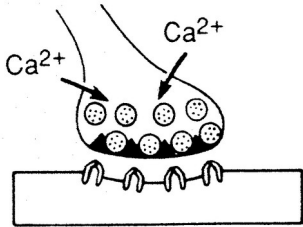
2D taxonomy of single cell models



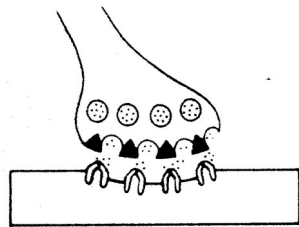
Synaptic models



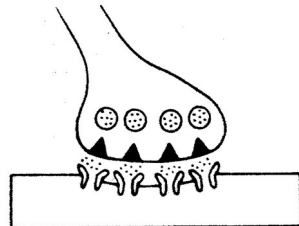
1. presynaptic action potential



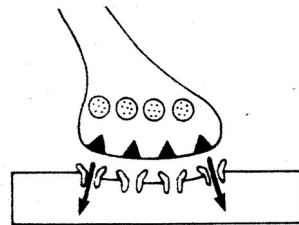
2. Ca^{2+} influx



3. transmitter release from the vesicles



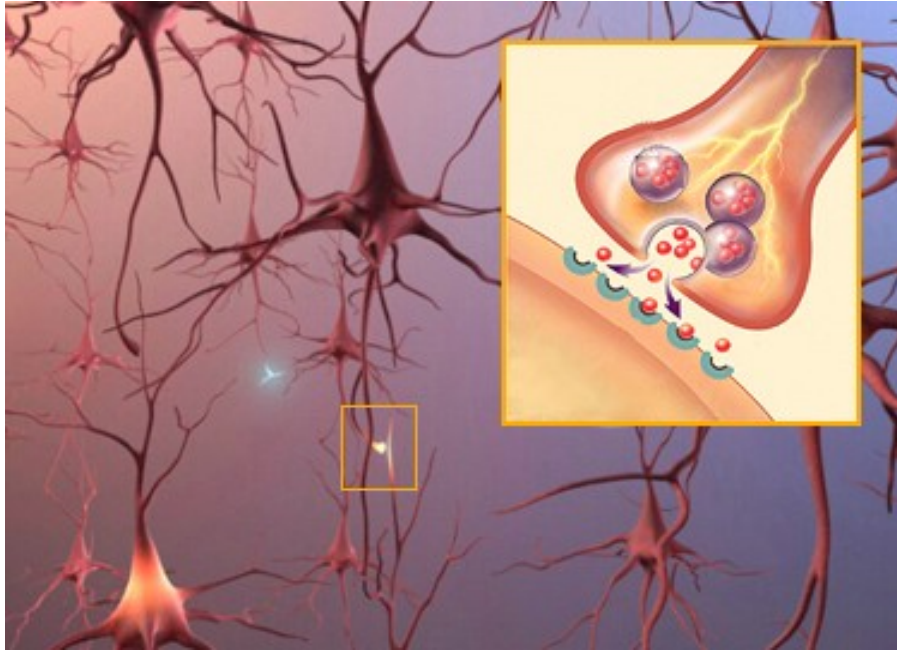
4. transmitter-receptor binding



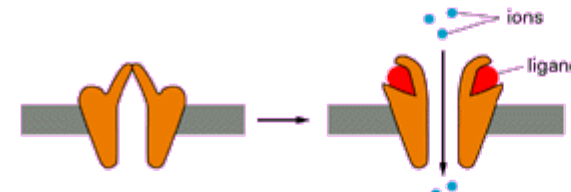
5. postsynaptic conductance ("PSG"),
current (PSC) and potential changes (PSP)

The aim synaptic models:
To calculate the postsynaptic
potential changes, based on
the presynaptic activity.

Between two neuron: The synapse

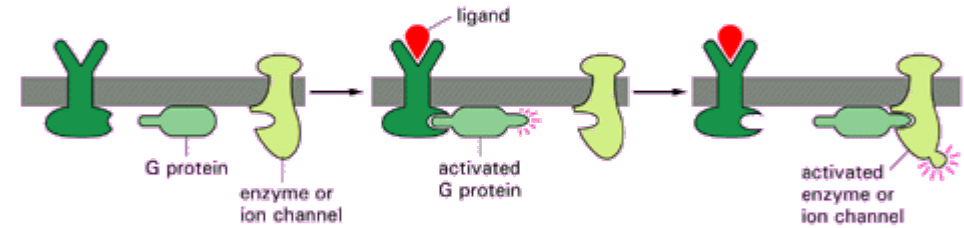


(A) ION-CHANNEL-LINKED RECEPTOR

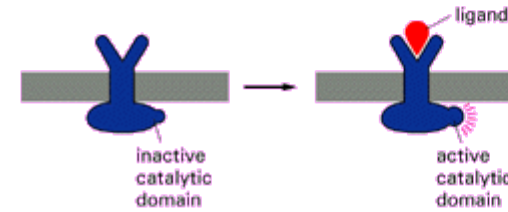


Ionotropic (A) and metabotropic (B,C) receptors

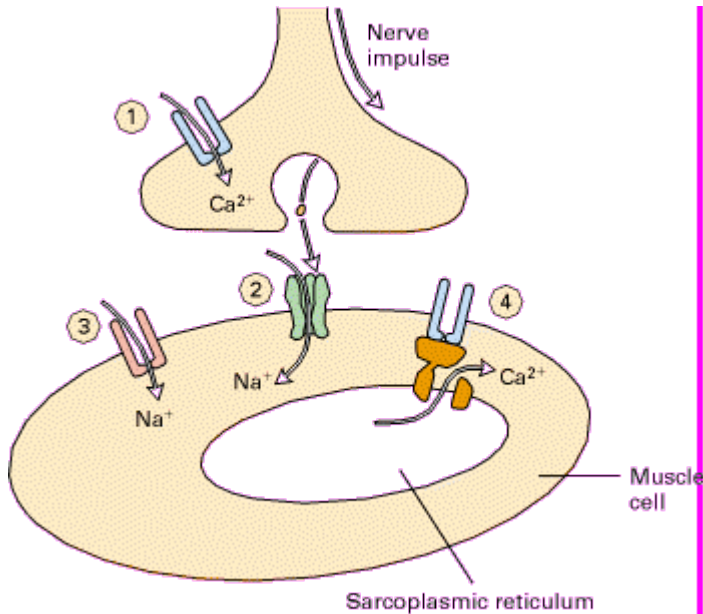
(B) G-PROTEIN-LINKED RECEPTOR



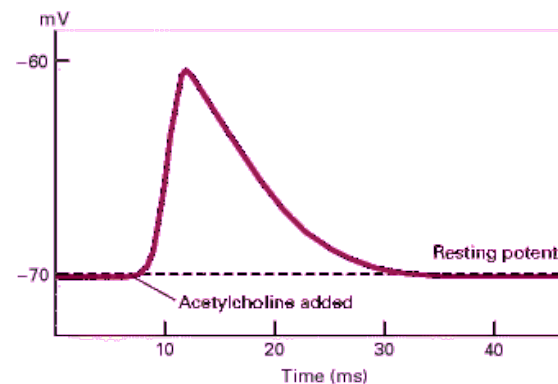
(C) ENZYME-LINKED RECEPTOR



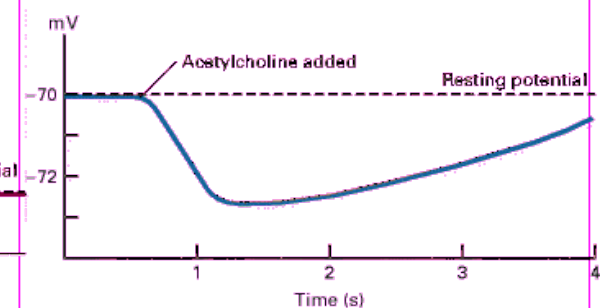
Excitatory and inhibitory postsynaptic potentials



(a) Excitatory synapse



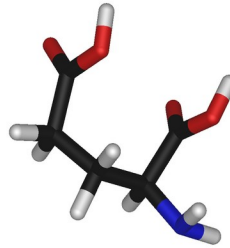
(b) Inhibitory synapse



Excitatory and inhibitory neurotransmitters

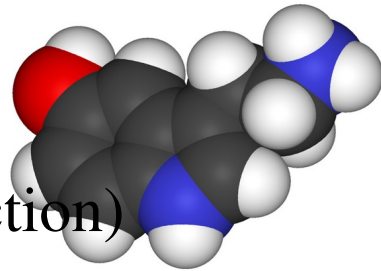
Glutamat

(information transmission)



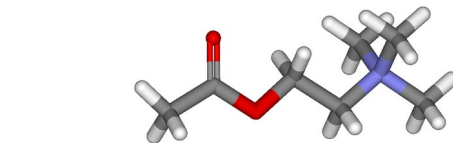
Serotonin

(mood, wake/sleep)



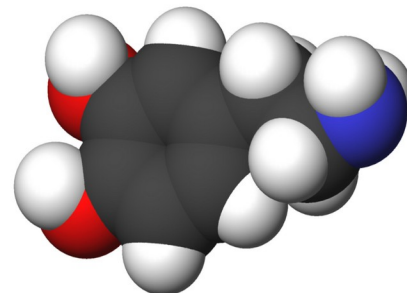
Acetylcholin

(neuromuscular junction)



Noradneraline

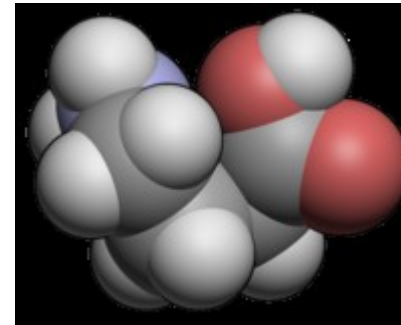
(arousal)



Dopamine

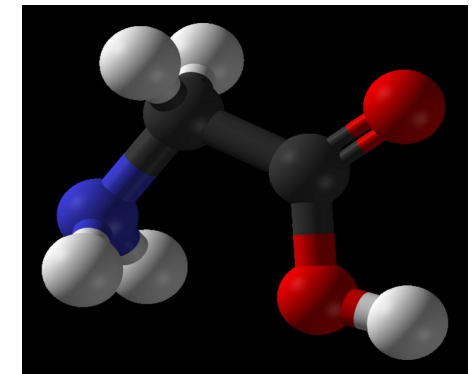
(reward system,
Parkinson disease,
schizophrenia)

GABA-gamma aminobutyric acid
(in the central neural system)



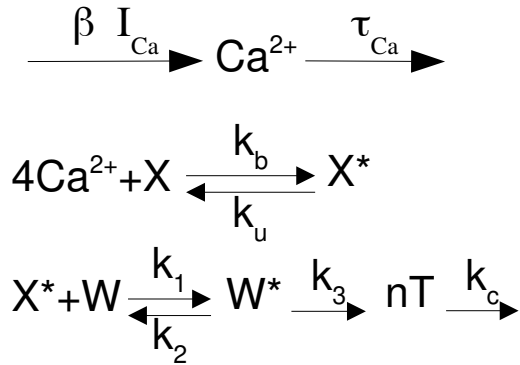
Glycine

(in the periphery)



Detailed kinetic synaptic models: the presynaptic side (1-3.)

kinetic schema (example)



Ca^{2+} intracellular calcium
 X, X^* protein, activated protein
 W, W^* vesicle, activated vesicle
 T transmitter

$$\frac{d[Ca^{2+}]}{dt} = \beta I_{Ca}(t) - \frac{[Ca^{2+}](t)}{\tau_{Ca}} - 4k_b[Ca^{2+}](t)X(t) + 4k_u X^*(t)$$

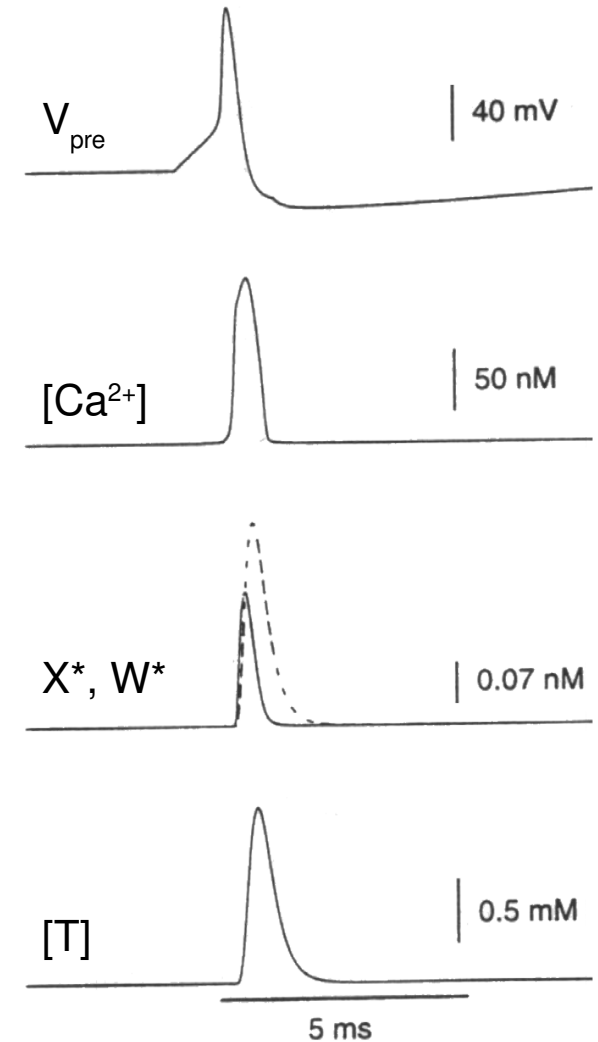
$$\frac{dX}{dt} = -k_b[Ca^{2+}](t)X(t) + k_u X^*(t)$$

$$\frac{dX^*}{dt} = k_b[Ca^{2+}](t)X(t) - k_u X^*(t) - k_1 X^*(t)W(t) + k_2 W^*(t)$$

$$\frac{dW}{dt} = -k_1 X^*(t)W(t) + k_2 W^*(t)$$

$$\frac{dW^*}{dt} = k_1 X^*(t)W(t) - k_2 W^*(t) - k_3 W^*(t)$$

$$\frac{d[T]}{dt} = k_3 n W^*(t) - k_c [T](t)$$

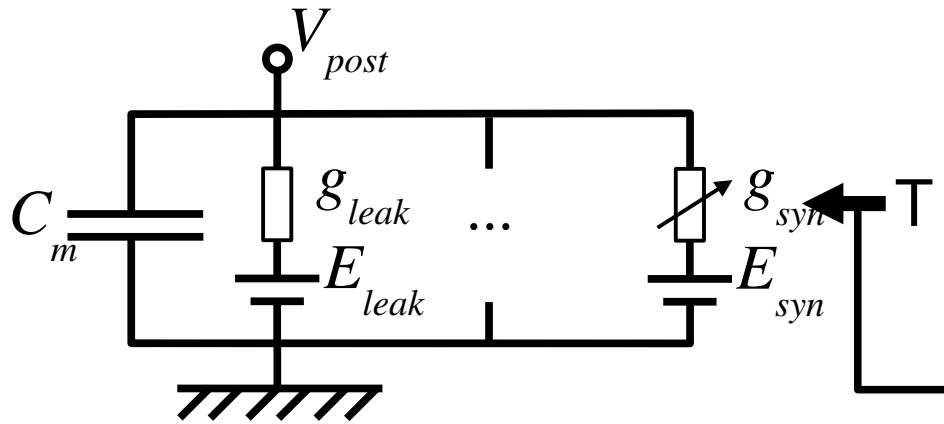


Reminder:

$$I_{Ca}(t) = \bar{g}_{Ca} s(t) (E_{Ca} - V_{pre}(t))$$

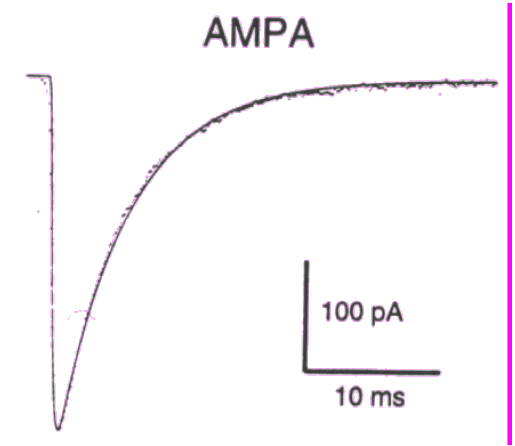
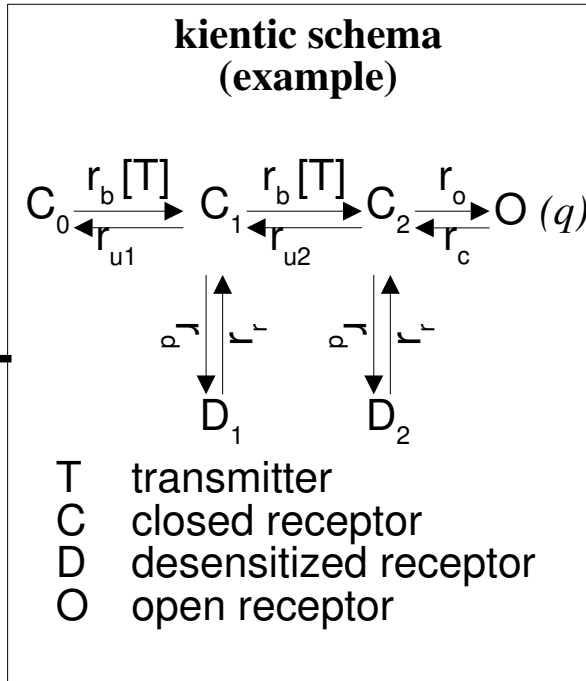
$$\frac{ds}{dt} = \frac{s_{\infty}(V_{pre}(t)) - s(t)}{\tau_s(V_{pre}(t))}$$

Detailed kinetic synaptic models: the postsynaptic side (4-5.)



Reminder:

$$I_{syn}(t) = \bar{g}_{syn} q(t) (E_{syn} - V_{post}(t))$$

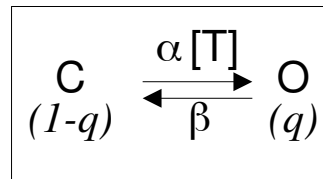


Simplified kinetic synaptic models

presynaptic side

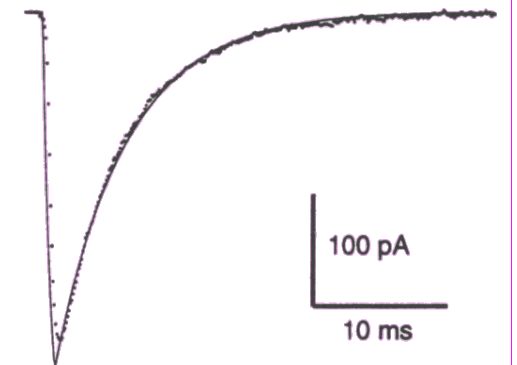
$$[T](V_{pre}(t)) = \frac{T_{max}}{1 + e^{-\frac{V_{pre}(t) - V_{\theta}}{K_{\theta}}}}$$

postsynaptic side



$$\frac{dq}{dt} = \alpha [T](t) (1 - q(t)) - \beta q(t)$$

AMPA

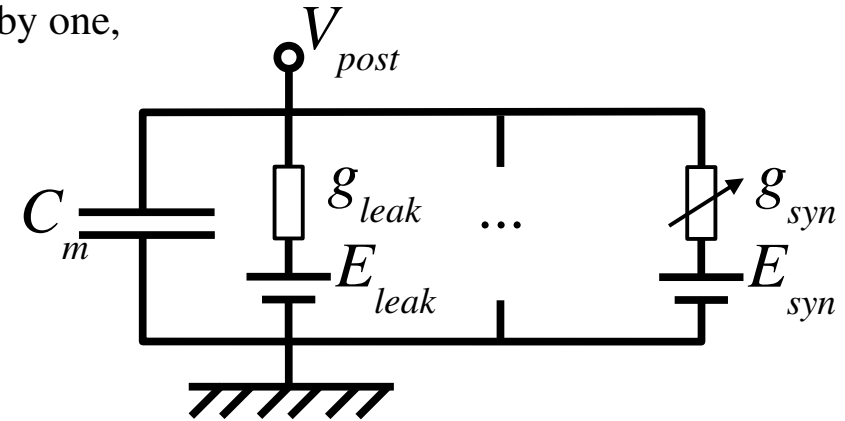


Phenomenological synaptic models

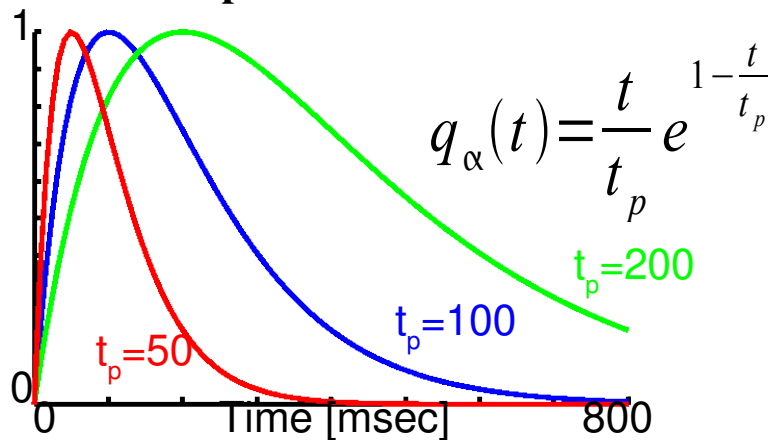
The postsynaptic conductance change, caused by one, single presynaptic action potential

$$I_{syn}(t) = \bar{g}_{syn} (E_{syn} - V_{post}(t)) \int_0^t \underbrace{\Delta(V_{pre}(t_0) - V_{\ominus})}_{\text{Linear summation of the individual conductance changes (convolution by Dirac-delta function)}} \cdot \overbrace{q(t-t_0)} dt_0$$

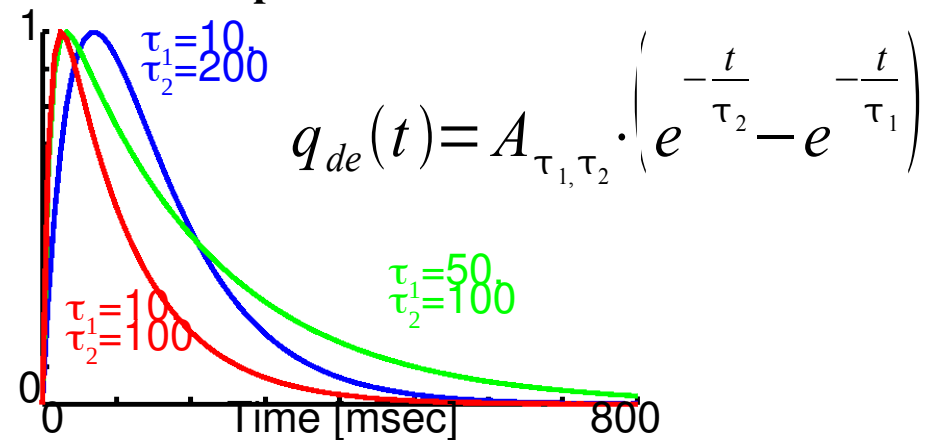
Linear summation of the individual conductance changes (convolution by Dirac-delta function)



alpha-function



double exponential function



Synaptic models: summary

		detailed	simplified	phenomenological
Number of variables	presynaptic	≥ 5	0	0
	postsynaptic	~ 5	1	
reproduced phenomena	senzitisation	yes	no	no
	desenzitisation	yes	no	no
	saturation (PSG, PSC)	yes	no	no
	saturation (PSP)	yes	yes	no

