“Navigare necesse est, vivere non est necesse.”

Pompeius

Navigation
What does navigation means?

„Navigation is the process of determining and maintaining a course or trajectory from one place to another. Processes for estimating one’s position with respect to the known world are fundamental to it. The known world is composed of the surfaces whose locations relative to one another are represented on a map.”


Thus, we need to know:

Where we are?
Where are the important places relative to me?
How to get there?
## Hierarchy of Navigation Strategies

<table>
<thead>
<tr>
<th></th>
<th>Information to be stored</th>
<th>External information required</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. Random Navigation</td>
<td>none</td>
<td>none</td>
<td>Random wandering</td>
</tr>
<tr>
<td>1. Praxic Navigation</td>
<td>Position in a sequence</td>
<td>none</td>
<td>Execution of a pre-defined / learned sequence of actions</td>
</tr>
<tr>
<td>2. Dead reckoning</td>
<td>Actual position</td>
<td>none</td>
<td>Path integration</td>
</tr>
<tr>
<td>3. Target approaching / Guidance / Avoidance</td>
<td>Sensible property of / landmark configuration at the aim</td>
<td>Direct / Indirect sensation of the aim</td>
<td>taxis</td>
</tr>
</tbody>
</table>
| 4. Place triggered   | 1. Landmark configurations defining places  
2. Local directional reference frames  
3. Direction of the movement, that leads to the aim | Actual landmarks | Self-localization by place recognition  
Association of recognized places to the actions that leads towards the aim |
| 5. Topological navigation | Set of landmark configurations linked by topological relationships | Actual landmarks | Graph searching, way finding               |
| 6. Metric Navigation | Set of landmark configurations linked by metrical relationships | Actual landmarks | Vector subtraction, trigonometrics          |
Typical praxic tasks and solutions

No external clue or sensory input is required
The Braitenberg cars: taxis
Place recognition-triggered versus topological navigation

(a) collection of routes to goals G1 and G2

(b) topological representation derived from the routes

Fig. 3. (a) With the place recognition-triggered response strategy there can be an ensemble of intersecting routes. The animat is able to go from S1 to G1, from S2 to G2, and from S3 to G1. However, if there is a new obstacle on the way from S1 to G1, as on this figure, the animat is lost, because the route from S1 to G1 is unique (see also Fig. 2). (b) In contrast, if the animat merges its representations of routes into a topological representation, the animat can go back to place A, take the sub-route between places A and B, and take the sub-route from place B to the goal G1. The resulting path is the concatenation of three sub-sequences, derived from three different routes.
Metric navigation

Detour finding

Shortcut finding

New wall

Unknown area

Unknown area

Known long route
Hippocampus

- Place recognition is required to apply higher order navigation strategies
- Place cells in the hippocampus (Pyramidal cells in CA1 and CA3 region, 1971)
Hippocampus

- Place recognition is required to apply higher order navigation strategies
- Place cells in the hippocampus (Pyramidal cells in CA1 and CA3 region, 1971)
Hippocampus in the human brain: episodic memory
What determines the position of a place field?

- Visual information
- but: in blind and deaf animals
- Tactile
- Olfactory
- Vestibular
- Memory traces
- Context
- Firing frequency is independent of the direction
- Independent of the aim
- Frequency coding

Phase precession – phase code

- Spatial code inside the place field?

Rat’s movement over time

Relation between firing and position

Action potentials of the place cell associated with PF1
Action potentials of the place cell associated with PF2

Hippocampal EEG
Theta cycle (transformed in a spatial scale)
Long Term Potentiation

- LTP – long term potentiation (1966): was discovered in the hippocampus.
Learning at cellular level

- LTD – long term depression: can be elicited by weak stimulus

- STDP – spike timing dependent plasticity:
Head direction cells

- Their firing correlate with the head direction
- Independently of the position
- During rest and movement
- presubiculum, postsubiculum, posterior cortex, thalamus, striatum

In the thalamus: The future head direction is coded (~25 ms latter), prospective coding.
Head direction cells

- Primary, based on external clues. (eq. visual)
- Without this, they are able to keep on the pattern, based on internal information (eq. vestibular)
- Removing the clue changes the firing pattern of all HD cells together by a random angle
- Furring of place cells change accordingly
Head direction cells

An attractor network model: ring of excitatory and inhibitory connections
Grid cells

- Pyramidal cells in the 2nd layer of the medial entorhinal cortex
- Position dependent activity
- Maximal firing frequency in the vertices of a triangular grid
- Parameters: spatial frequency, orientation, 2D phase

Hafting et al., Nature, 2005
A new type of spatial representation: The grid cells

Action potentials  Firing frequency  Autocorrelation

Hafting et al., Nature, 2005

- Pyramidal cells in the 2nd layer of the medial entorhinal cortex
- Position dependent activity
- Maximal firing frequency in the vertices of a triangular grid
- Parameters: spatial frequency, orientation, 2D phase

How does a crystal-lattice get to the brain?
The basic properties of the grid system

Location and the structure of the MEC

Grid periodicity increases along the dorso-ventral axis
The basic properties of the grid system

Phase: The neighboring cells have common period length, but their grid is shifted.

The grid pattern appears for the first run.
Firing properties of the grid cells

Independent of the size of the environment

Determined by visual clues

Immediate appearance

Fyhn et al., 2004
Hafting et al., 2005
Nobel prize in medicine 2014

The 2014 Nobel Prize in Physiology or Medicine

John O'Keefe
Born 1939, USA
University College London

May-Britt Moser
Born 1963, Norway
Norwegian University of Science and Technology, Trondheim

Edvard I. Moser
Born 1962, Norway
Norwegian University of Science and Technology, Trondheim

John O'Keefe, May-Britt Moser, and Edvard I. Moser were awarded the 2014 Nobel Prize in Physiology or Medicine for their discoveries of cellular mechanisms underlying navigation and spatial memory.
Possible role: path integration
A Spin Glass Model of Path Integration in Rat Medial Entorhinal Cortex Mark C. Fuhs and David S. Touretzky (Journal of Neuroscience)
How it is possible to determine the position based on grid code?

By summing up grids with corresponding phase

This corresponds to a Chinese remainder numeral system. Unique until the least common multiple.
The capacity increases exponentially

Yoram Burak, Ted Brookings, Ila Fiete (arXiv)
Models
Fig. 24. Firing phase computation yields $\alpha$, the angle between the heading direction of the animat and the direction defined by the position of the animat and the centroid of landmarks 1 and 2; the phase will be “Late” if $|\alpha|$ is smaller than 60°, “Middle” if $|\alpha|$ is between 60° and 120° and “Early” if $|\alpha|$ is greater than 120°. (After Burgess et al., 1994.)
Reaching the aim, the animat looks around into all the 8 directions.
Figure 8: Typical firing rate maps of cells in the different layers of the model after 60s exploration, showing qualitative agreement with known extracellular recordings from entorhinal cells and place cells, and predicting firing rate maps for subicular cells and 'goal cells'. The simulated rat moves evenly across the environment; spikes were binned in a $10 \times 10$ grid, each contour represents 10% of the peak firing rate. Top row: entorhinal cell, peak rate is 40Hz (left), place cell peak rate is 30Hz (right). Bottom row: subicular cell, peak rate is 40Hz (left), goal cell representing east (the goal is at the centre), peak rate 101Hz (right).
Figure 5: One possible set of firing rate maps giving rise to a population vector representing the position of the rat from the goal at G.
Summary

- One-shot learning at goal
- Hebbian learning
- Latent learning
- Phase-presession
- Sensory neurons?
- Goal cells?
- PC and SC layers are unnecessary
- Works only in a limited distance from the goal (SC place field size)
Reinforcement learning: an actor-critique architecture

FIGURE 2. The actor-critic system. a: An input layer of place cells projects to the critic cell, C, whose output is used to evaluate behavior. Place cells also project to eight action cells, which the actor uses to select between eight possible directions of movement from any given location. b: An example of a Gaussian place field (x and y axes represent location, z axis represents firing rate).
Parallel learning of policies and the values
The temporal difference rule:

The critic depends on position $p$:

$$C(p) = \sum_i w_i f_i(p)$$

The value function ($V$) must satisfy:

$$V(p_t) = (R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \ldots)$$

Where $\gamma$ is a constant discount factor for predicted (not actual) reward. From this the consistency of the value:

$$V(p_t) = \langle R_t \rangle + \gamma V(p_{t+1})$$

A well trained critic should satisfy the same consistency assumption:

$$C(p_t) = \langle R_t \rangle + \gamma C(p_{t+1}).$$

The actual difference between the two sides governs the learning, this is called the temporal difference learning rule:

$$\delta_t = R_t + \gamma C(p_{t+1}) - C(p_t)$$

The weight changes are proportional the difference:

$$\Delta w_i \propto \delta_t f_i(p_t).$$
The temporal difference rule:

Two sources of value:

\[ \delta_t = R_t + \gamma C(p_{t+1}) - C(p_t) \]

The actually achieved reward

The difference between the expected and the achieved increase.

Together expresses the difference between the expected and actual reward.
The effect of reward in dopaminergic cell of basal ganglia

An interpretation:

Dopamine cells signals the difference between the expected and received reward.
This article describes a computational model of the hippocampus that makes it possible for a simulated rat to navigate in a continuous environment containing obstacles. This model views the hippocampus as a ``cognitive graph'', that is, a hetero-associative network that learns temporal sequences of visited places and stores a topological representation of the environment.

Calling upon place cells, head direction cells, and ``goal cells'', it suggests a biologically plausible way of exploiting such a spatial representation for navigation that does not require complicated graph-search algorithms. Moreover, it permits ``latent learning'' during exploration.

The model implements a simple ``place-recognition-triggered response'' navigation strategy. It implements and uses fine details as phase precession and spike time dependent plasticity.
Animat navigation using a cognitive graph

Phase precession explained by sequence learning. Each bin in this grid-like world corresponds to a unique place. The rat has learned the sequence of places from A to J. It subsequently moves from A to J (top): when it is in A, it recalls the sequence from A to G (bottom); when it is in B, it recalls the sequence from B to H; and so on. Each movement and each prediction phase takes a full theta cycle. Thus, the representation of the current place starts a new theta cycle and the prediction of place E comes earlier and earlier in the cycle (dotted arrow), that is the phase of &ring; of the place cell corresponding to E diminishes
The structure of the model
Olivier Trullier, Jean-Arcady Meyer: *Biol. Cybern.* 2000
The modified connection between two place cells in neural space corresponds to the facts that the corresponding placefields are neighbors and that the place field of the post-synaptic cell is in the direction corresponding to the head-direction that modulates the connection, with respect to the place field of the pre-synaptic cell.
the synaptic weight between a given place cell and a given goal cell is an inverse function of the distance from the goal to the preferred location of the place cell.
Exploitation

The synaptic weight between a given place cell and a given goal cell is an inverse function of the distance from the goal to the preferred location of the place cell.
Signal propagation from the goal location towards the east.
Activity fields of the eight goal cells
Creating subgoals

Resulting trajectories
Possible alternative routes
Spatial cognition and neuro-mimetic navigation: a model of hippocampal place cell activity

Angelo Arleo, Wulfram Gerstner

Centre for Neuro-Mimetic Systems, MANTRA, Swiss Federal Institute of Technology Lausanne, 1015 Lausanne, EPFL, Switzerland

Fig. 1. Functional overview of the model. Allocentric and idiothetic stimuli are combined to yield the hippocampal space representation. Navigation is based on place cell activity, desired targets, and rewards

Fig. 8. The variance of the sEC cell activity around the center of mass \( p_{sec} \). When the variance falls below the fixed threshold \( \Sigma \) the spatial location \( p_{sec} \) is used to calibrate the robot’s position

Fig. 9. Uncalibrated dead-reckoning error (curve a) versus calibrated robot positioning using sEC cell activity (curve b)

Fig. 11. a Two-dimensional view of the environment with a feeder location (dark grey square) and two obstacles (white rectangles), and an example of robot trajectory induced by the action cell activity after learning. b Vector field representation of the learned navigational map
Extraction of distance from grid cell activity
Analytic calculations based on grid decomposition
Number of necessary cells to represent distances up to a given precision

The number of necessary cells increases as power low with the size of the environment in case of simpler solutions, only grid cells provide scalable solution, which increases logarithmically.
• The rat is exploring a 20 x 20 m arena

• When it reaches the significant place (origin), grid cell synapses get potentiated

• One-shot learning: synaptic weight becomes proportional to the firing frequency of the (presynaptic) grid cell at the origin

• The animal's distance from this point should be measured
RESULTS

Input from grid cells

Spacing: 2.9 m  Spacing: 4.6 m  Spacing: 7.3 m

Firing pattern of distance cells

MODEL A

Firing pattern of distance cells

MODEL B
Why it is important?

The main drive of the mammalian evolution was the conquer of the night
The path integration based navigational system of the rat works well in the darkness
Navigation in the darkness requires awareness of non-sensible objects
This could be a main step towards higher level of abstraction
Thank you!