Data analysis methods in the neuroscience

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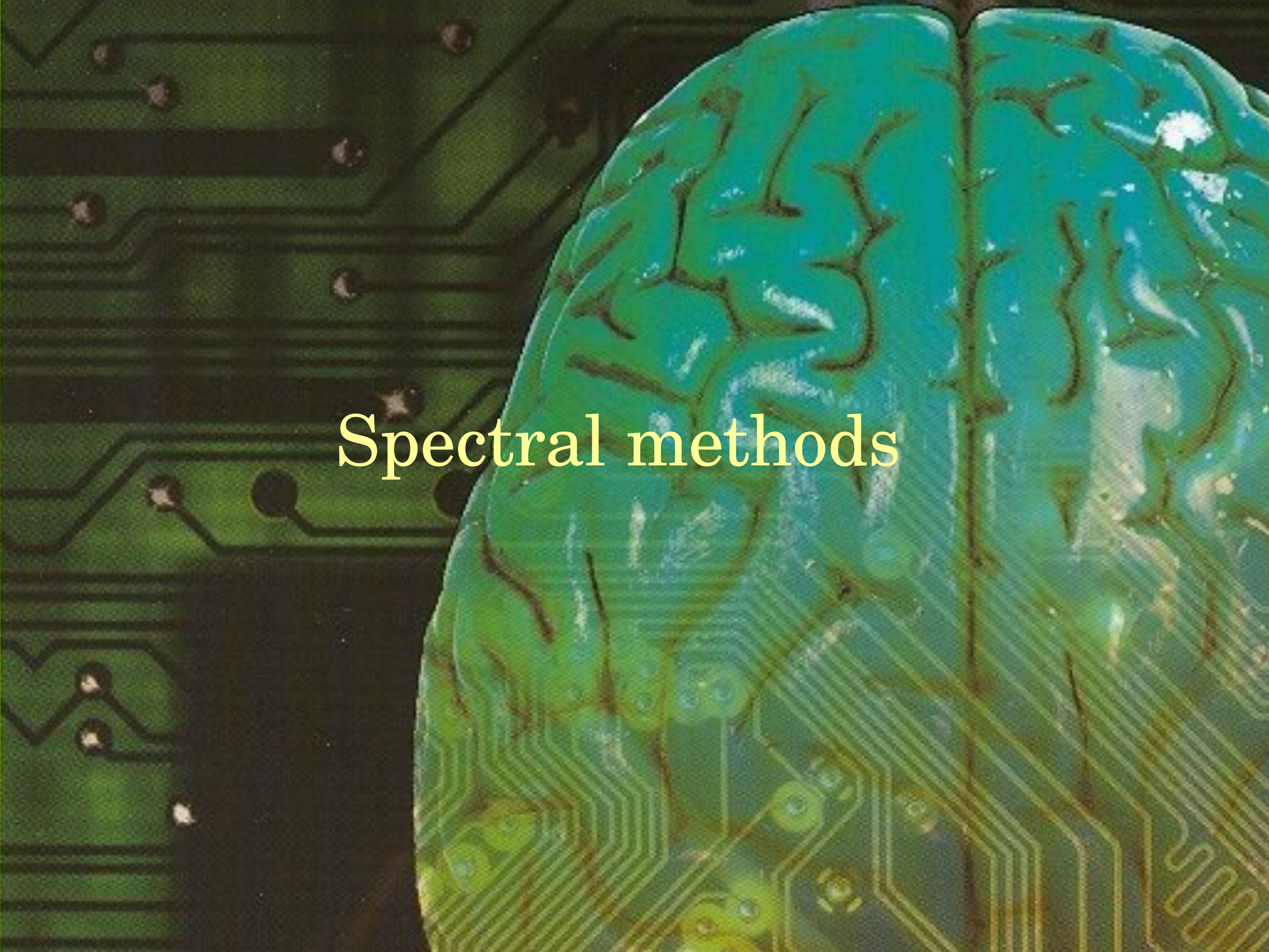
Spectral methods

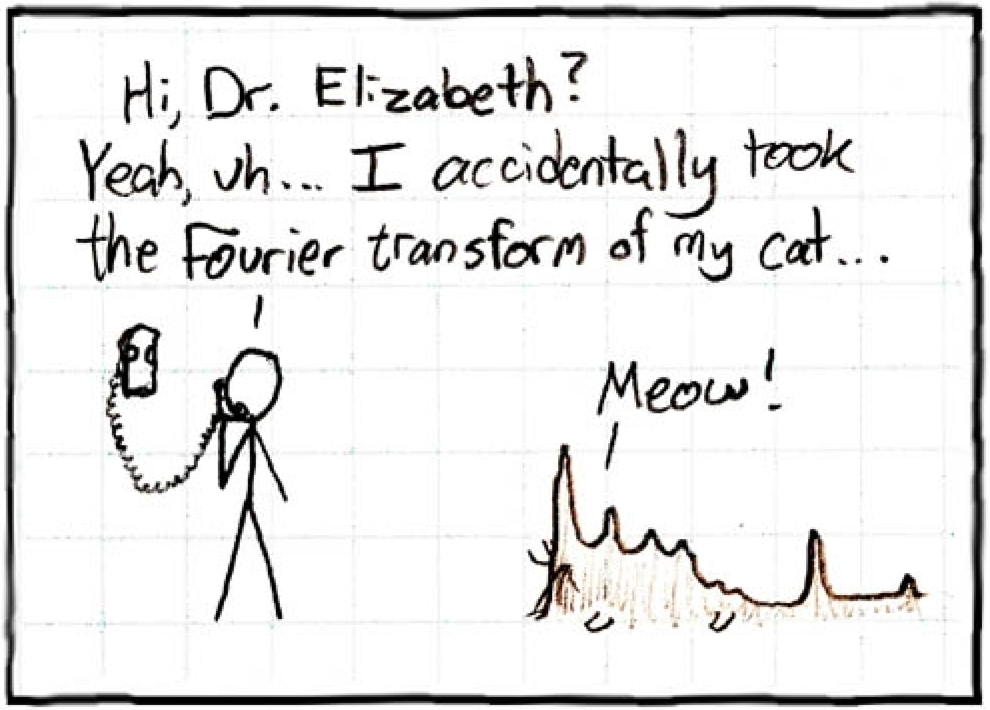


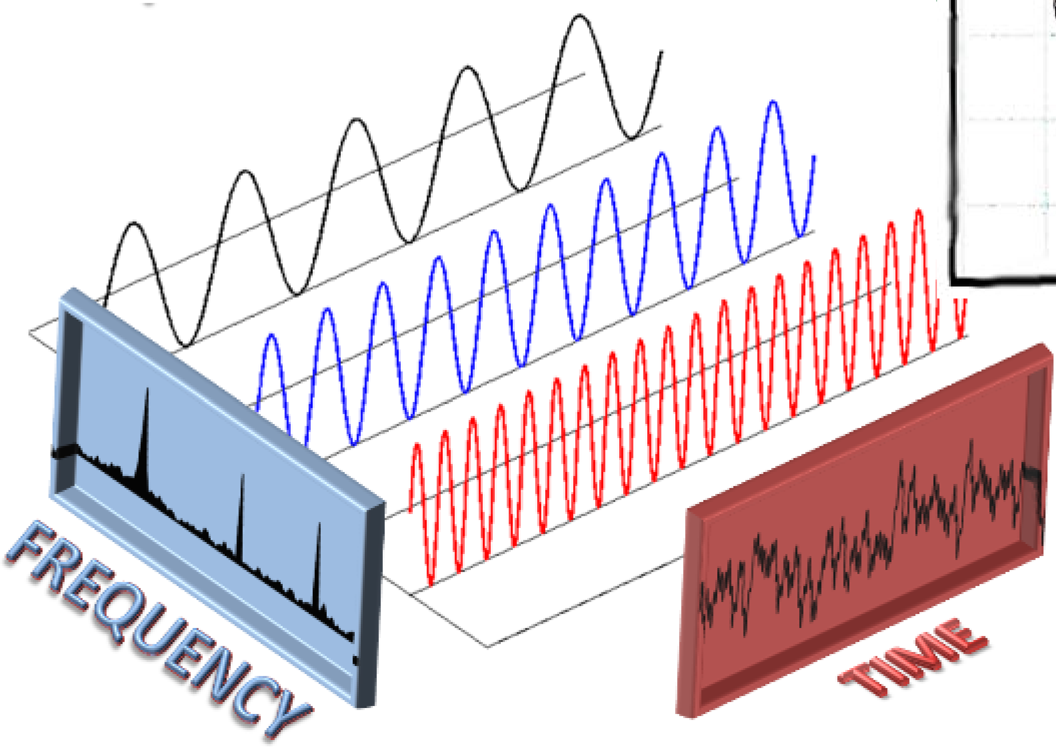
Methods applicable to one time series

The Fourier transformation

$$g(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$= \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$





The Fourier transformation

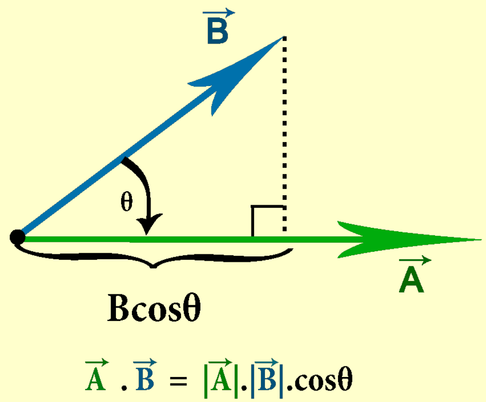
$$\bar{A}\cdot\bar{B} = AB\cos\theta = (A\cos\theta)B = A(B\cos\theta)$$

$$i\cdot i = j\cdot j = k\cdot k = 1; \quad i\cdot j = j\cdot k = k\cdot i = 0;$$

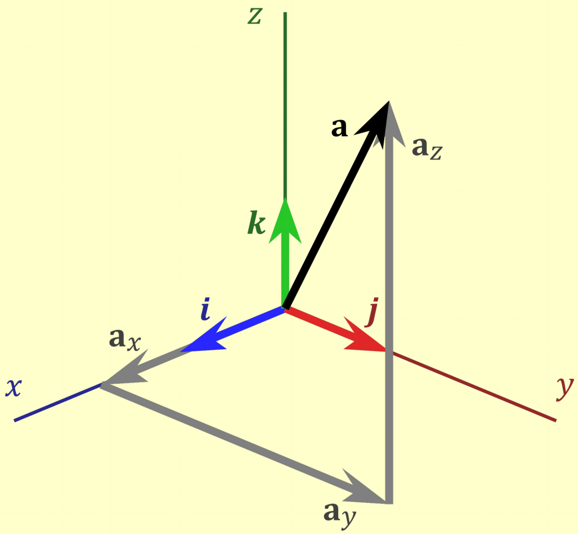
$$\bar{A}\cdot\bar{B} = (A_x i + A_y j + A_z k)\cdot(B_x i + B_y j + B_z k)$$

$$= A_x B_x + A_x B_x + A_x B_x$$

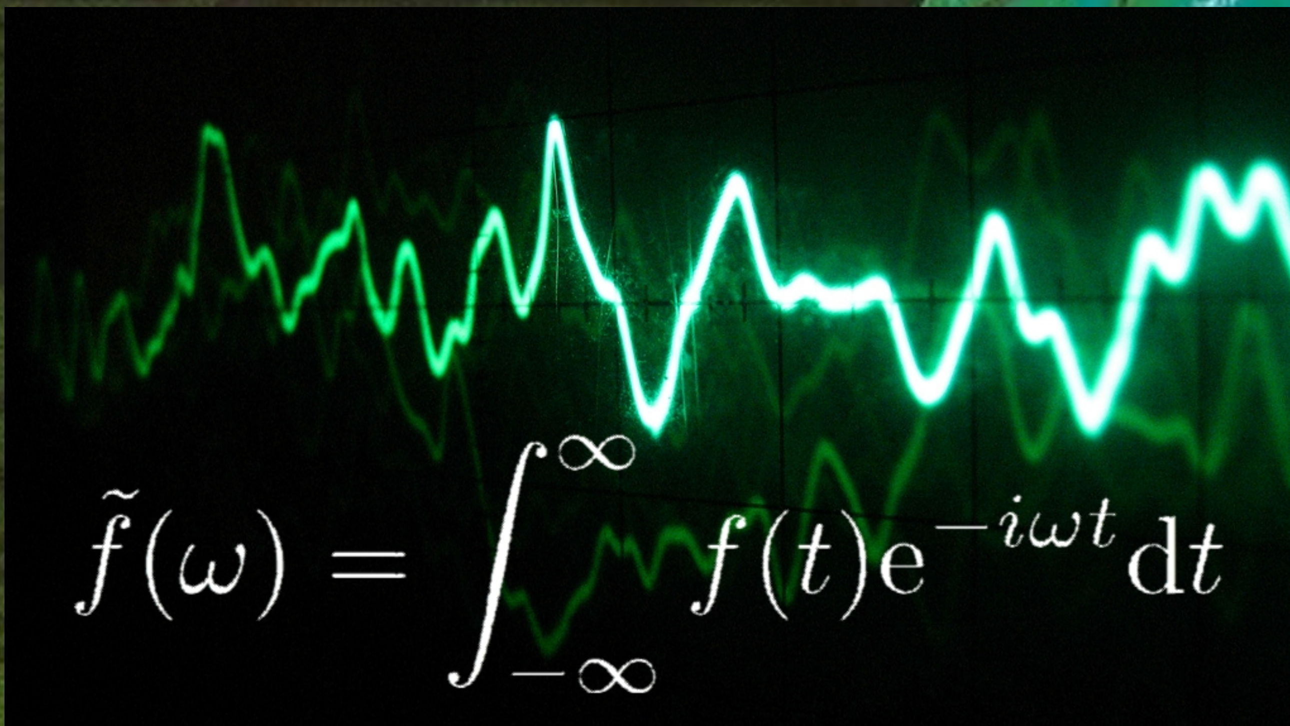
$$\bar{A}\cdot\bar{B} = AB(\hat{e}_A \cdot \hat{e}_B) = AB(1)(1)\cos\theta = AB\cos\theta$$



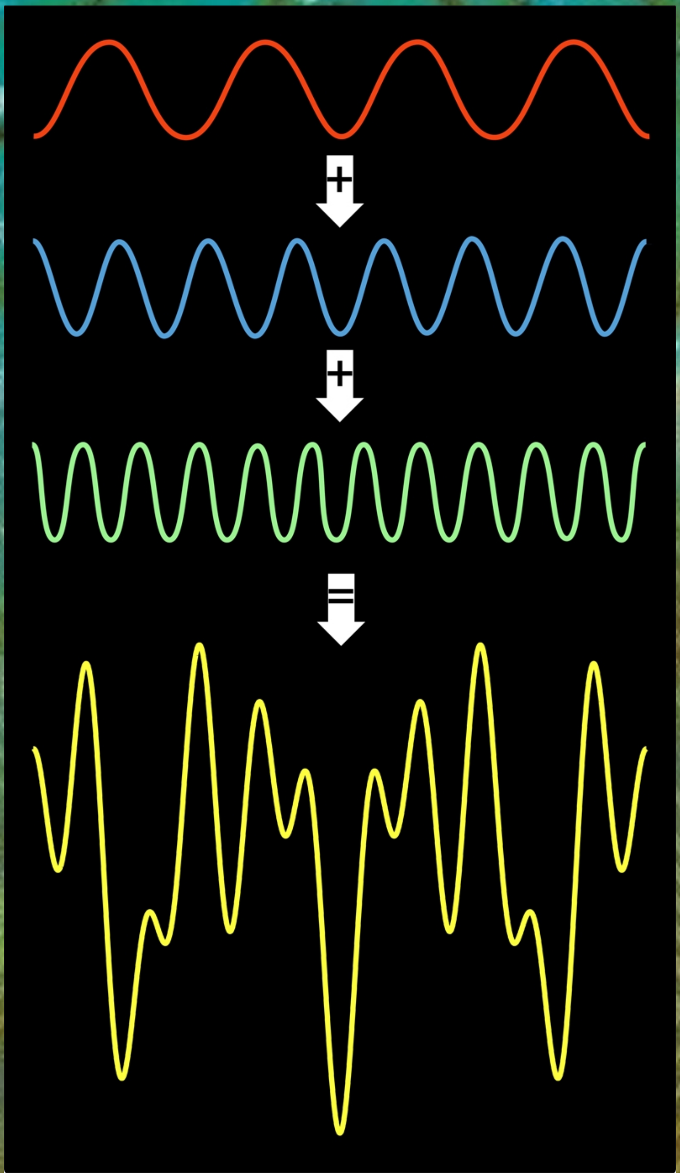
Coordinates: projection (dot product) onto the orthogonal unit vectors (base) of the coordinate system



The Fourier transformation

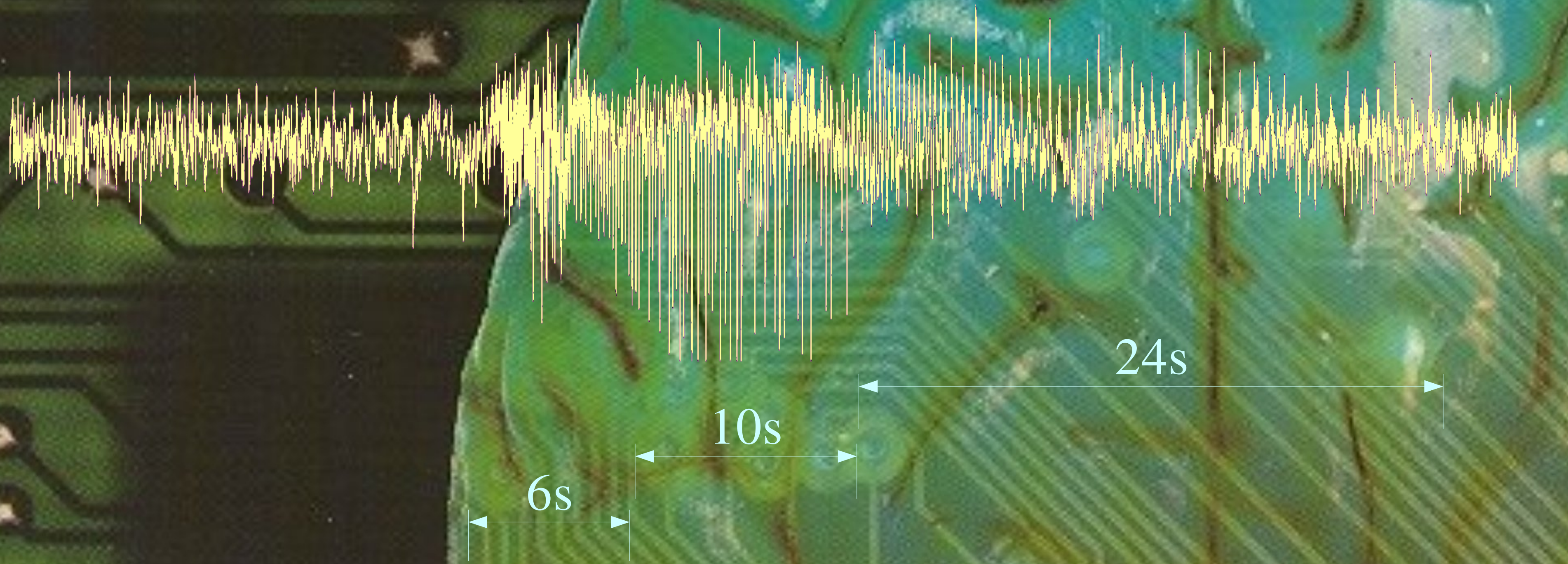


$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t)\mathrm{e}^{-i\omega t}\mathrm{d}t$$



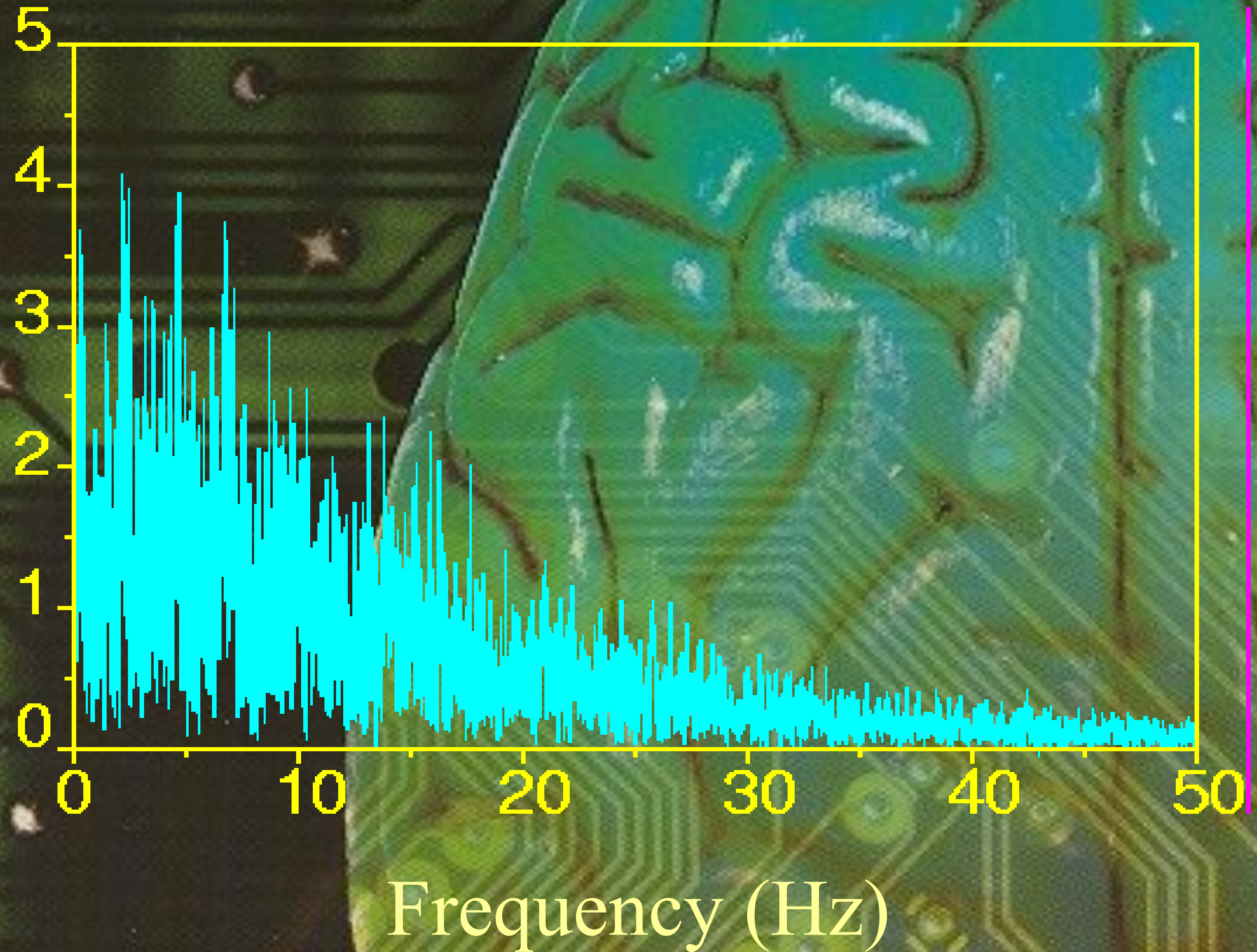
Example: Slow dynamics of the epileptic seizure

An experimental epilepsy model: Generalized epilepsy evoked by local application of 4-Aminopyridin, ECoG:



Three phases of the seizure can be distinguished, based on amplitudes, frequencies and waveforms.

The Fourier spectrum



The Fourier spectrum

What about the frequency axis? How do we know, which spectrum element conrresponds to which frequency?

We need the sampling frequency: F, measured in Hertz.
The length of the Fourier spectrum is equal to the length of the original data set: N samples
The total length of the recording in sec is T=N/F

The N-th spectrum line corresponds to the sampling frequency: F
Note: the spectrum is meaningfull only until F/2, the Nyquist frequency.
F/2 is the maximal frequency which could be mesured by F sampling frequency.

Thus the frequency step, or the unit of the frequency axis is F/N=1/T

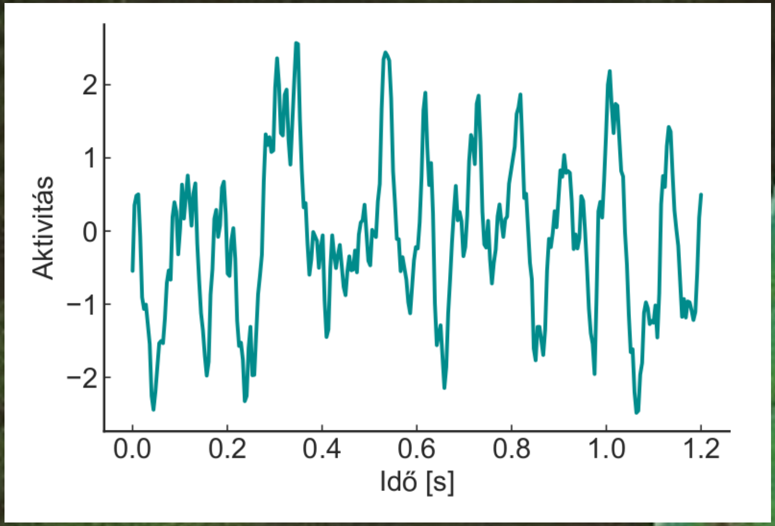
The Fourier spectrum

Fine details:

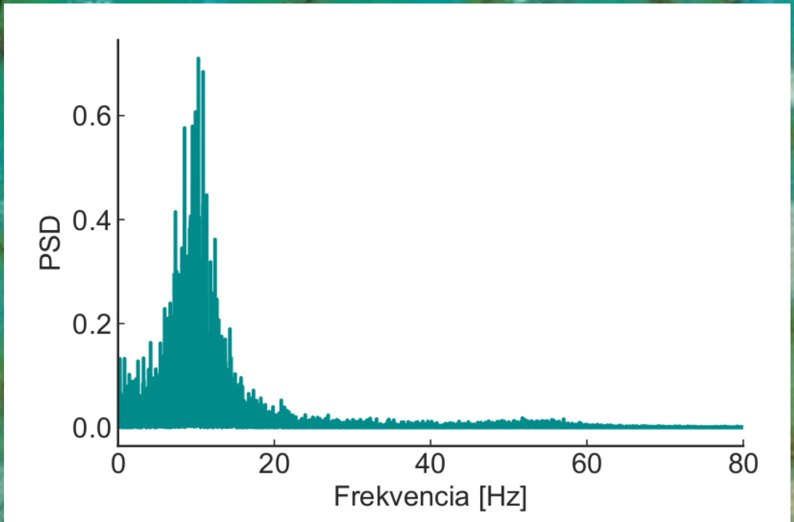
- The results of the FFT algorithm is a vector of complex numbers of length N.
- Real part corresponds to the cosine, the imaginary part for the sine functions. From their ratio, a phase can be calculated for all frequencies.
- The square of the absolute value is the power spectrum.
- The first element of the spectrum is the 0 frequency, the offset constant or mean of the data series. It breaks the symmetry, as it only appears at the lower end of the spectrum.
- The real part of the rest N-1 element is symmetrical, the imaginary part is antisymmetrical.
- The frequencies above N/2 are also called negative frequencies, and can be drawn from -F/2 to 0.
- For data series consist of even samples, the Nyquist frequency (F/2) appears only onece in the middle of the spectrum, while for odd samples if appears twice.

How to get a smooth Fourier spectrum?

Data



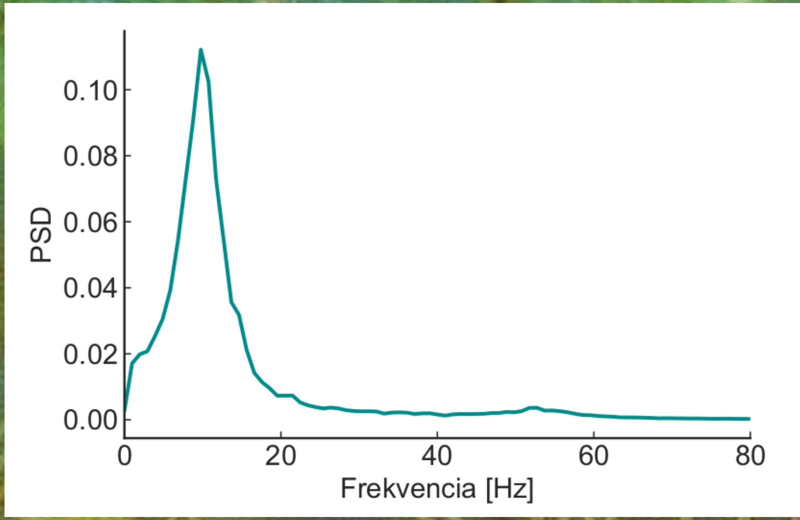
Standard Fourier spectrum



Welch method:
Cut into shorter pieces and average the spectra.

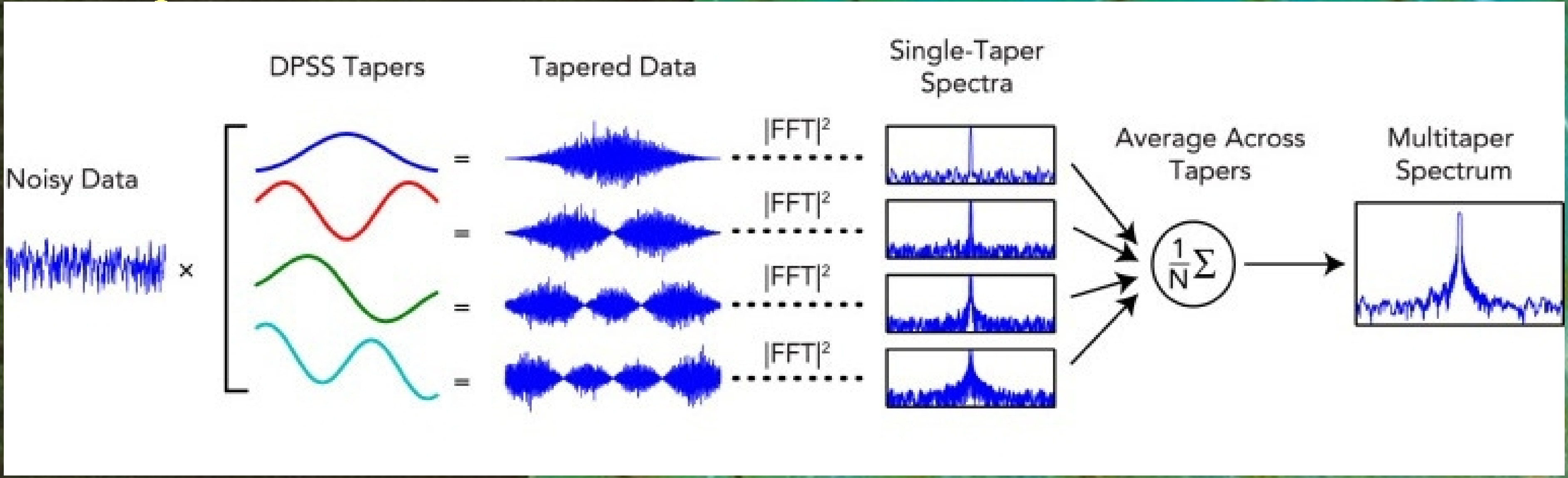
The specrum became smoother, but the frequency resolution get worse.

Welch method

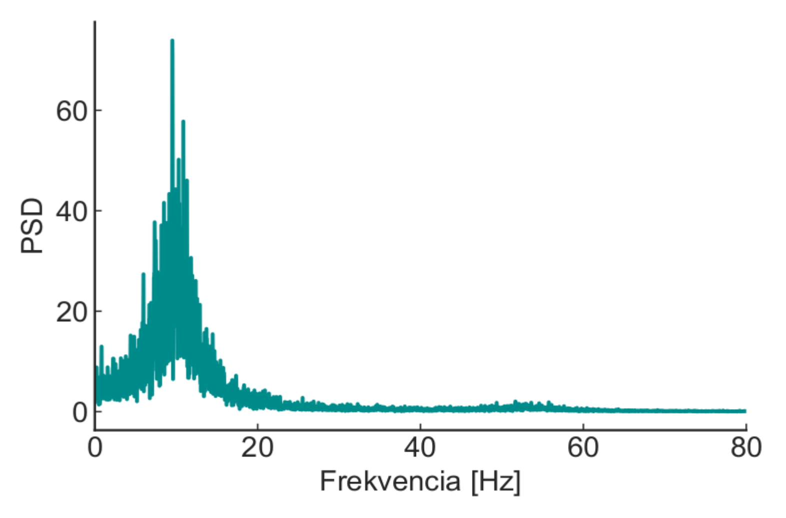


How to get a smooth Fourier spectrum?

Multitaper method



Multitaper spectrum



Apply long and orthogonal frames to the data and average the resulted specra.

The frequency resolution preserved and the spectrum get smoother.

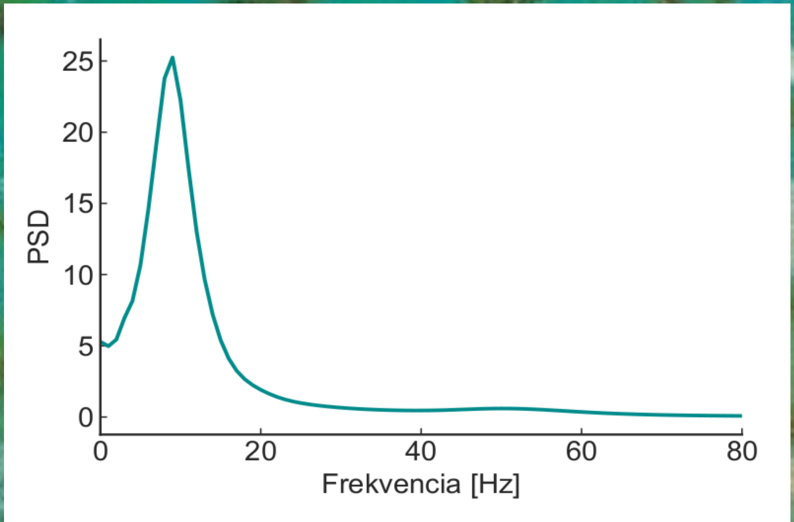
How to get a smooth Fourier spectrum?

The best result can be calculated from the wavelet analysis.

The specrum is smmoth and the frequency resolution is adaptive: better for the lower frquencies and gradually decaying with the inscreasing frequencies.

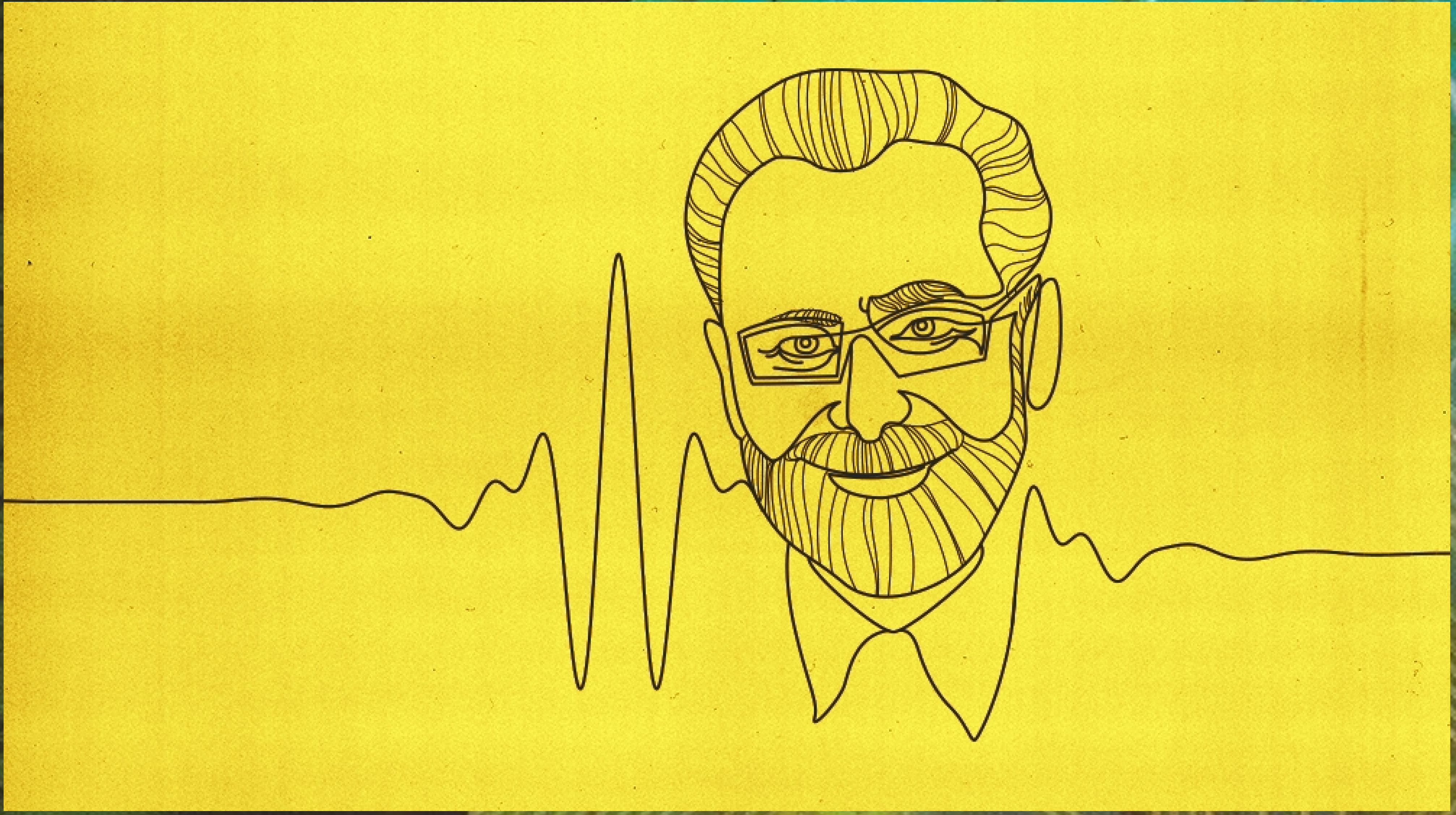
As it is calculated from the Wavlet coefficiences, lets see the wavelet transformation before!

Wavelet spectrum



Wavelet-transformation

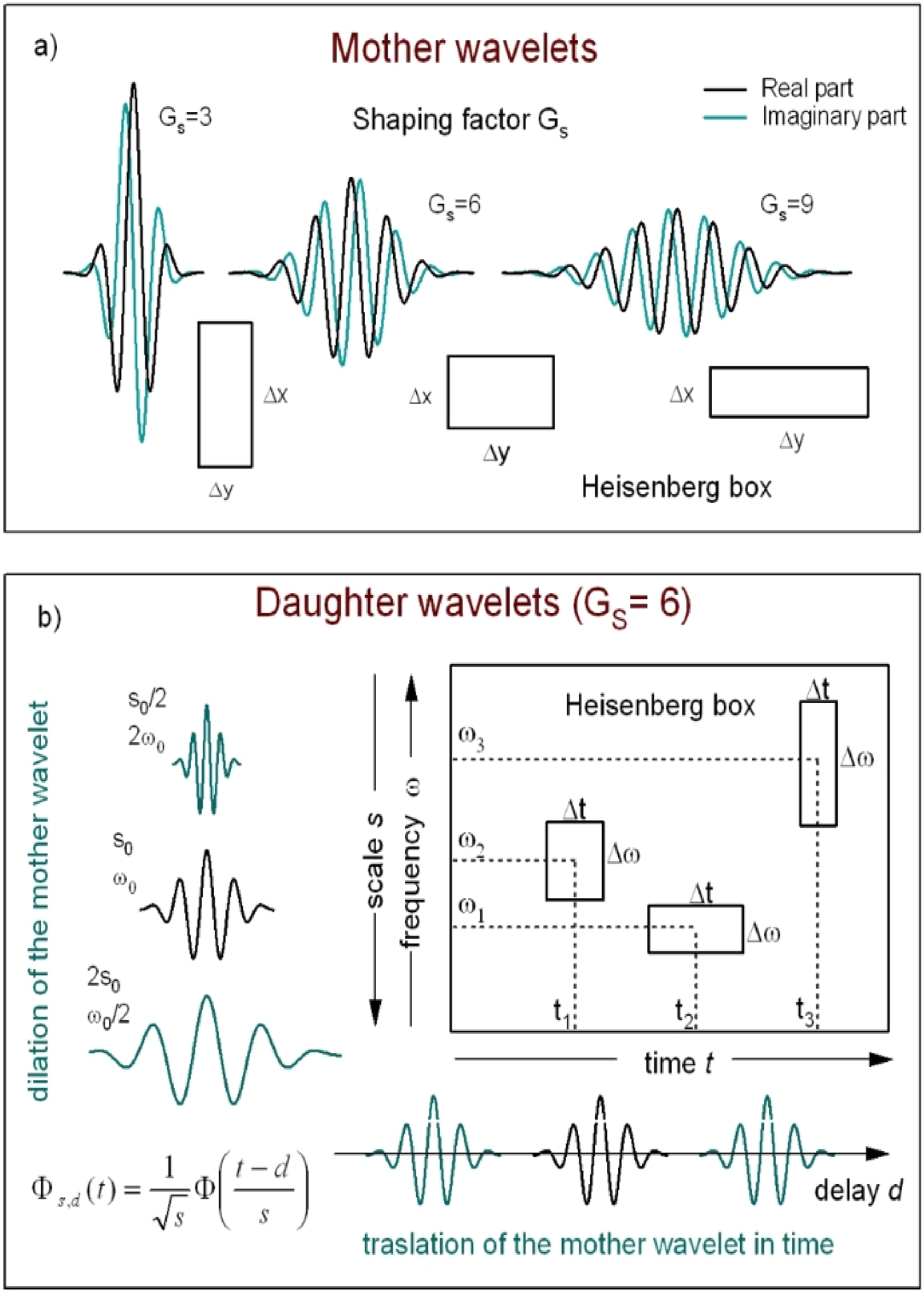
Ywes Meyer Abel-prize 2017



Wavelet-transformation

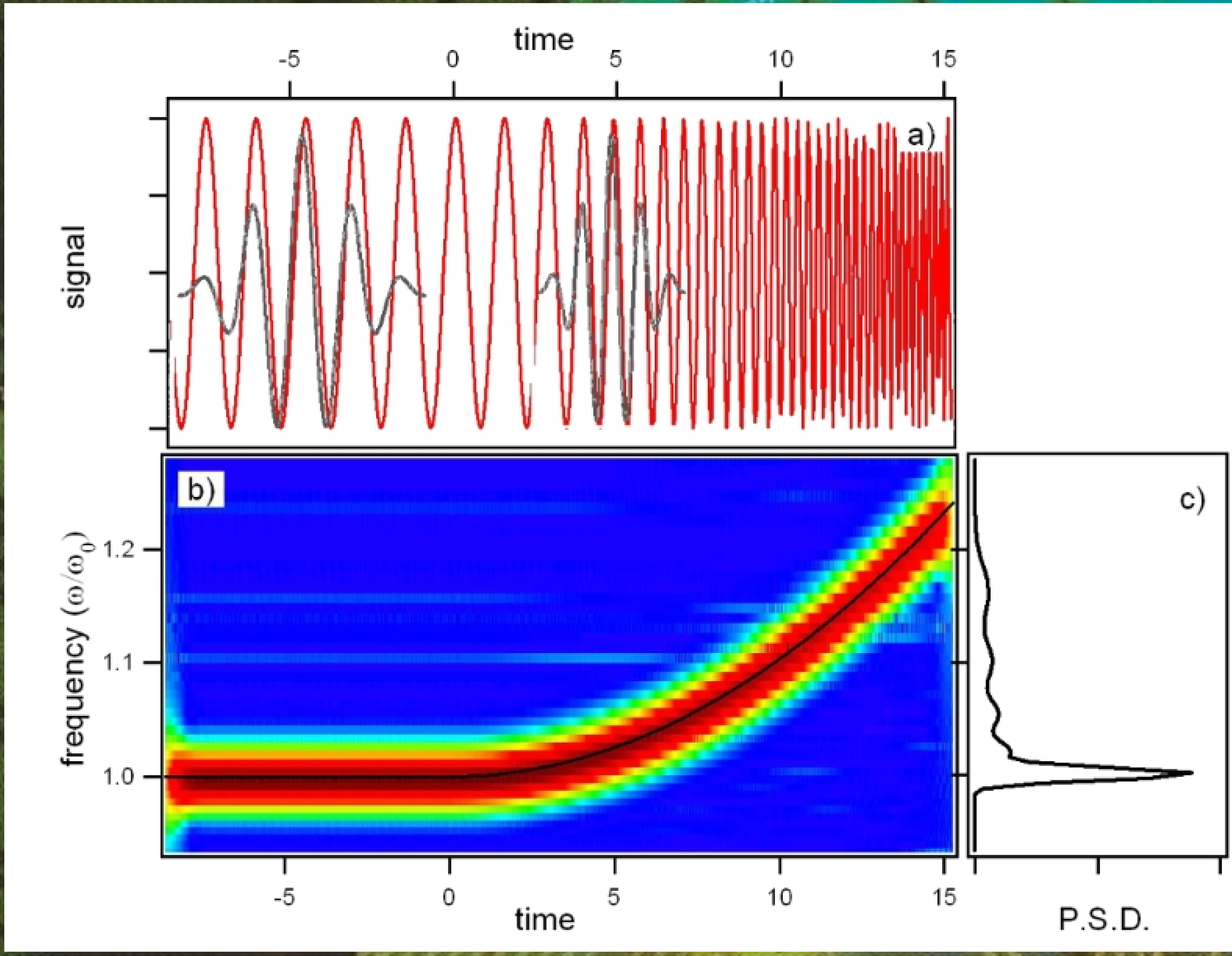
Both time and frequency resulution is preserved. The data is decomposed into wavelet components, which are confined both in time and in frequency.

The temporal and frequency resolution is adaptive.



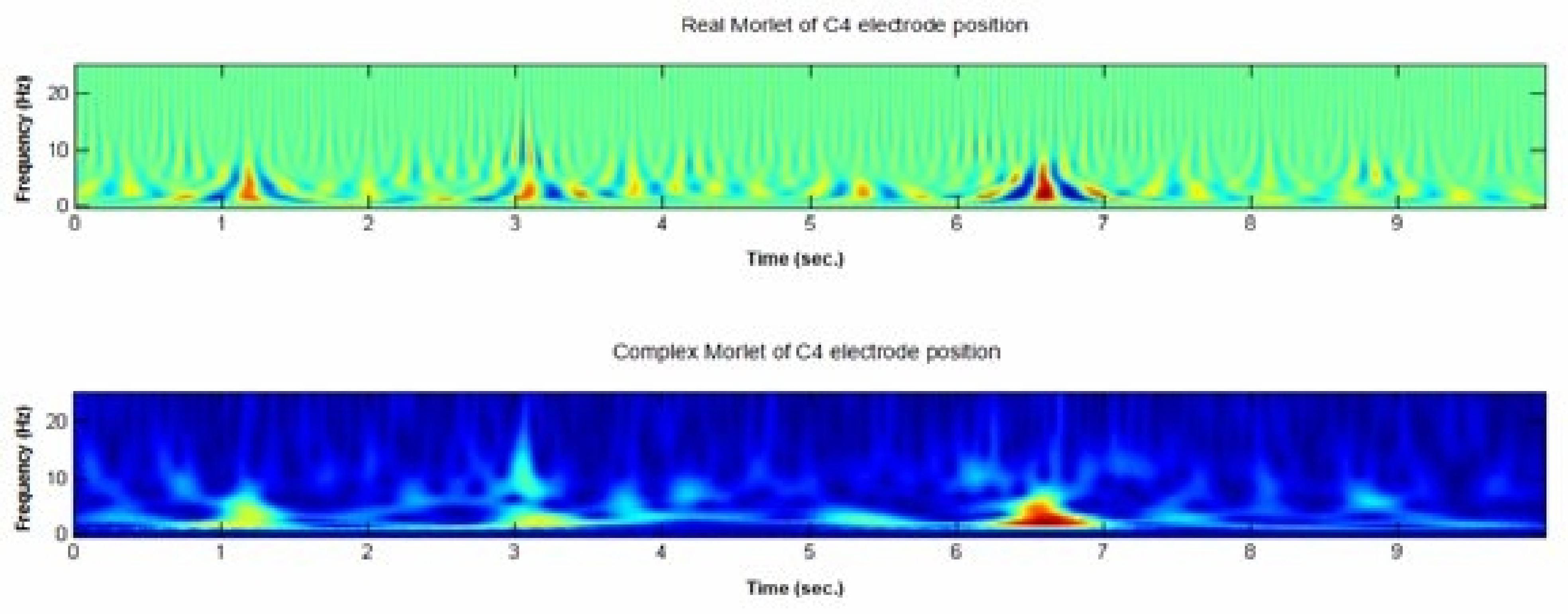
$$\Phi_{s,d}(t) = \frac{1}{\sqrt{s}}\Phi\left(\frac{t-d}{s}\right)$$

Wavelet-transformation

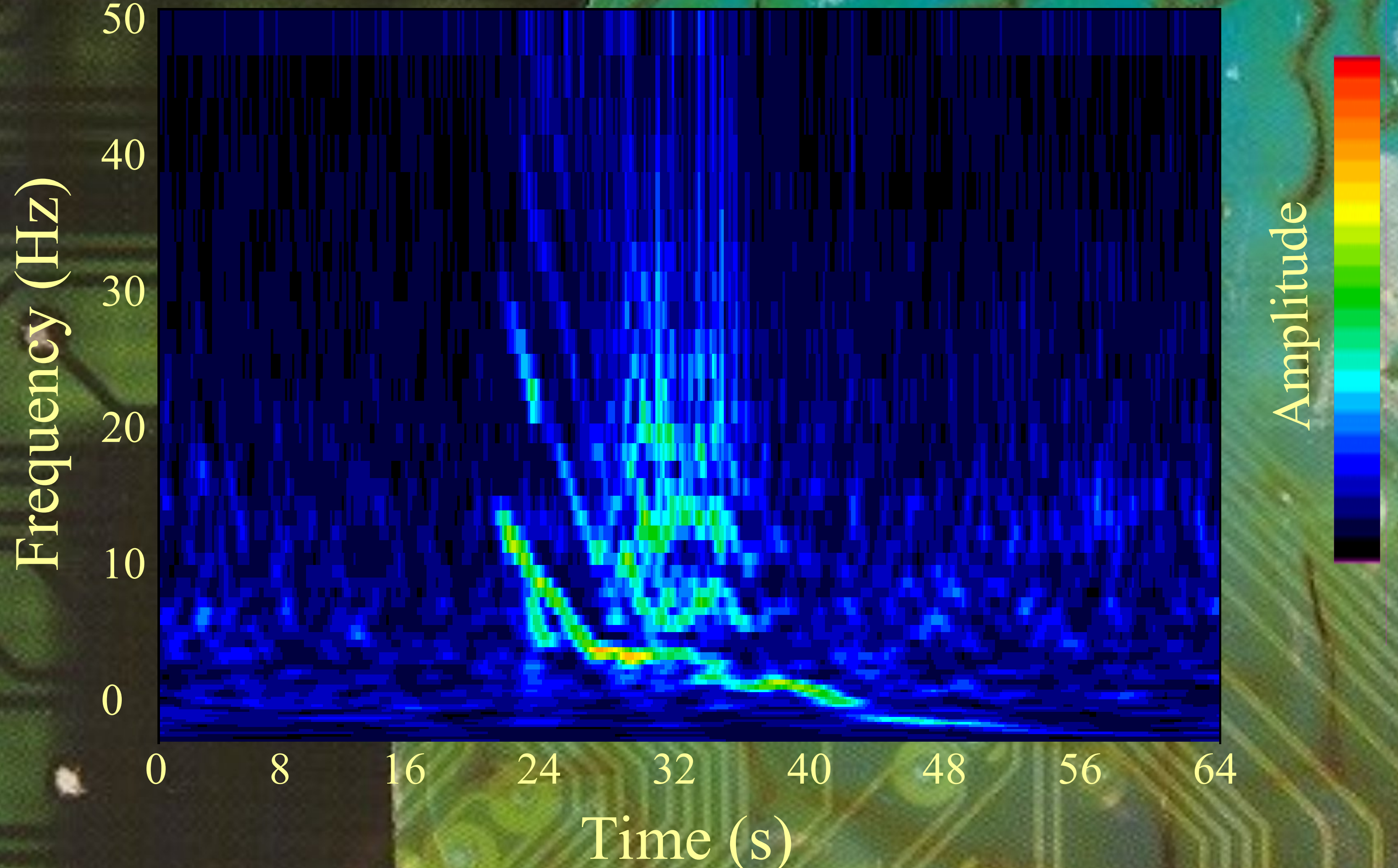


Wavelet-transformation

Real wavelet amplitude vs complex wavelet amplitude



Wavelet-transformation of the ECoG



Matching pursuit algorithm

Matching with predefined dictionary of extended “wavelets” of different kind:

Monotonous tones, point spikes, Gabor wavelets, etc.