Causality analysis for brain research

Zsigmond Benkő[1 2], Zoltán Somogyvári [1]

[1]Wigner Research Centre for Physics of the HAS, Budapest, [2]Semmelweis University, Budapest;

Introduction
The vast amount of neural data opened a new era of brain research where new data analysis methods are highly needed for taking a full advantage of the resources we have. Such methods for example the causality detection methods, which try to extract causal relations from data based on observations without any experimental intervention.

Aim
In this poster we attempt to review mainstream causality analysis methods.

Methods
In the first part we present methods based on Norbert Wiener's notion of causality on stochastic systems. We start with the first formalization of the Wiener-principle, namely Granger causality. Then we continue with the exact information-theoretical formulation (Schreiber) by introducing Transfer Entropy, which measures predictive information transfer between two variables. In the second part we present state-space methods. This approach applies for deterministic dynamical systems and based on Takens theorem. In one hand we show how Convergent Cross-Mapping works and review enhanced versions of it since of its invention in 2012. The other hand we present how complexity and causality are related.

Results
We show the methods in work on simulation examples, discuss the capabilities and draw the borders of applicability.

Conclusion
The stochastic and deterministic dynamical system views are complementary approaches and the simultaneous application of the two to neural data bears the promise of a new level of understanding of the information processing and the underlying dynamics.

Fundings: Hungarian Scientific Research Grant (OTKA) K113147
benko.zsigmond@wigner.mta.hu

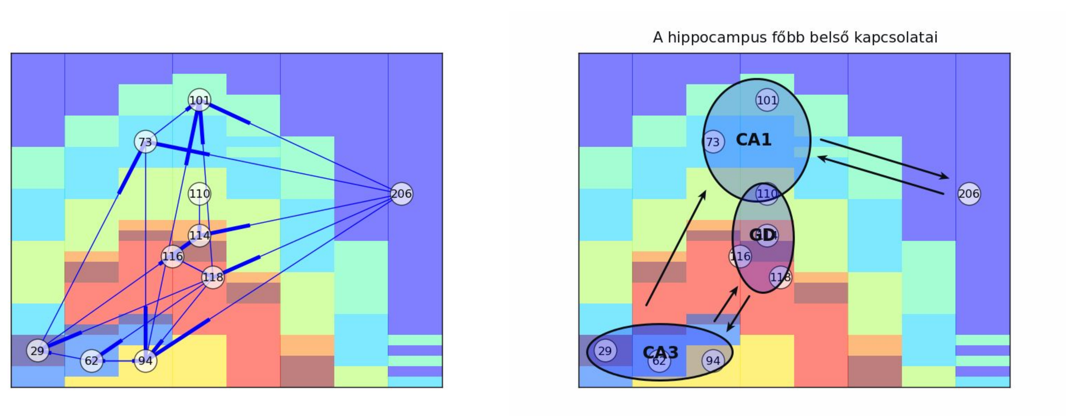
Granger causality

According to Norbert Wiener a variable X is a cause of an other varibale Y if one can make better predictions on the future values of Y based on not just past Y values but including the past values of X too.
The first formalization of this idea was made by Clive Granger in the framework of linear autoregressive models:

$$y_t=\sum_{j=1}^{m} a_j y_{t-j}+\epsilon_{1t} \qquad y_t=\sum_{j=1}^{m} b_j y_{t-j}+\sum_{j=1}^{n} c_j x_{t-j}+\epsilon_{2t}$$

$$F_{X\rightarrow Y}=\ln\left(\frac{var(\epsilon_1)}{var(\epsilon_2)}\right)$$

If X Granger-causes Y then the error variance of the extended model is significantly smaller than the more concised model's error variance.



Granger causality can be applied in the frequency domain, and in this case the causal relations can be seen by spectral components.
The method needs relatively few data-points, and the estimation process is faster compared to non-parametric methods, but the linearity and stationarity assumptions restrict the interpretation of the results. A bigger problem of Granger causality is that in the case of circular causality the predicted directionality of the connection becomes unreliable.

Transfer Entropy

Shannon entropy quantifies the uncertainty over some quantity:

$$H(X)=-\sum p(X_i)\ln(p(X_i))$$

The conditional entropy quantifies the uncertainty when some other variable is observed:

$$H(X|Y)=-\sum p(Y_j)\sum p(X_i|Y_j)\ln(p(X_i|Y_j))$$

Mutual information quantifies the information gained by observing y variable:

$$I(X,Y)=H(X)-H(X|Y)$$

We can define conditional mutual information by:

$$I(X,Y|Z)=H(X|Z)-H(X|Y,Z)$$

A non-parametric translation of Norbert Wiener's original idea to information theory's language is Transfer Entropy introduced by Thomas Schreiber in 2008. Transfer Entropy quantifies the predictive information transfer, the Mutual Information between future Y values and present X states conditioned on present Y states.

$$TE(X\rightarrow Y)=I(Y_{t+1},X_t|Y_t)$$

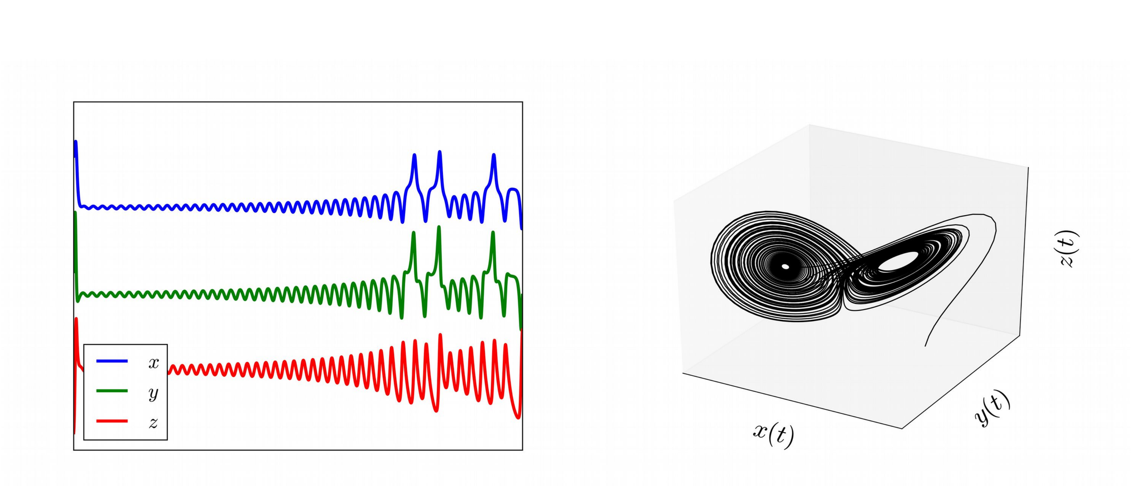
Transfer Entropy and Granger causality are equivalent in the case of jointly gaussian variables. TE was used to reconstruct interaction delays in turtle visual system (Wibral et al., 2013).
There are several toolboxes for the computation of TE, for example JIDT with python bindings or TRENTOOL for matlab. The method is data and computation intensive and has the same problems as Granger causality except linearity.

Convergent Cross-Mapping

A dynamical system view of causality detection was invented by George Sugihara in 2012. A dynamical system is a system whose state (Z) changes with time. From the current state of a system one can predict all the coming future states, if time evolution rules are known.

$$Z_t=\phi_t(Z_0)$$

The time-evolution can be stroboscopic like in the case of discrete maps or continuous like in the case of flows. An example for discrete chaotic map is the logistic map, and an example of a continuous flow is the Lorenz-system.

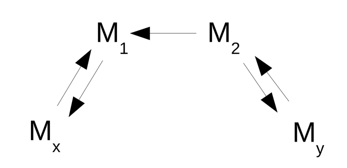


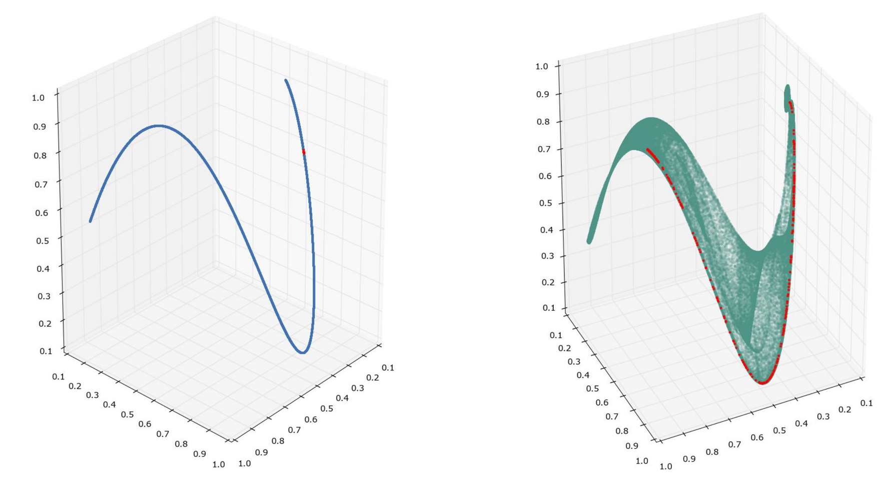
The actual state is a point in state space, which space is spanned by the state variables as axes. As time evolution goes on the system's state traces out a trajectory in state space defined by the actual values of state variables in every time instance. In many cases the system's state gets attracted to a lower dimensional subspace of state space and the points form there a manifold.
Sugihara's idea is based on Takens theorem which claims that the state of a chaotic dynamical system can be restored (reconstructed) with the aid of one time series measurement from that system by a process called time delay embedding. The method has two parameters the embedding dimension (d) and the embedding delay (τ).

$$X_t=(x_t,x_{t-\tau},x_{t-2\tau},\ldots,x_{(d-1)\tau})^T$$

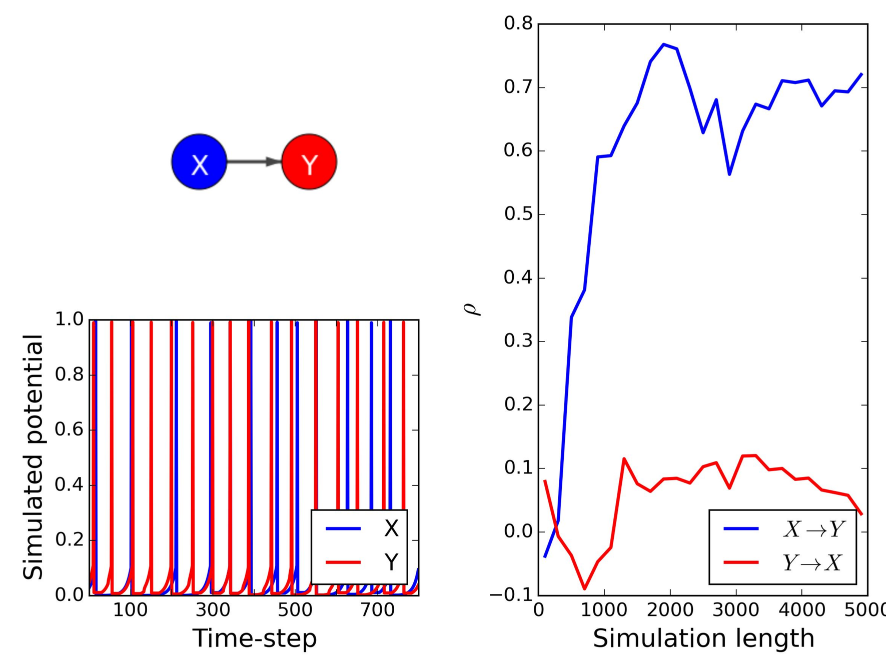
According to Takens theorem the time delay procedure at d>= 2m+1 is an embedding that is for every point there is an invertible smooth mapping (whose derivative is also injective) between the reconstructed and the original state space.
From here follows that the manifold formed by the points in the reconstructed state space is topologically equivalent with the manifold in the original state space, meaning that every point has the same set of neighboring points in both spaces.
Sugihara et al. generalized Takens theorem and then looked at the picture in a multivariate way. The basic idea is that if two time series measurements (X,Y) were from the same dynamical system then the reconstructed state spaces can be mapped back to the same original manifold, so there should be a smooth mapping between them too. In this case one can say there is circular causality between the two variable.

Asymmetrical relation arise when the original state-spaces are not the same, but one of them is a lower dimensional projected version of the other. The mapping works in one direction, but this operation is non-invertable. In this case one can speak about unidirectional causal relation, where the mapping works from the effect to cause but not the other direction.

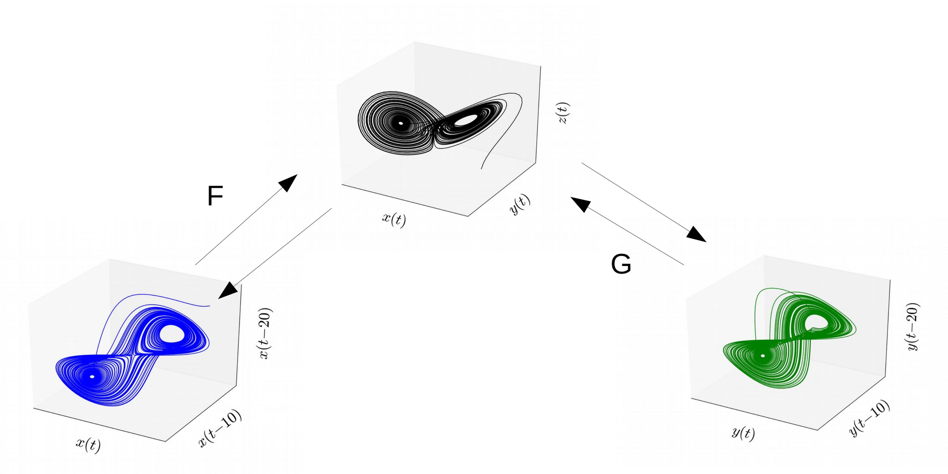




If there are no such mappings between the two reconstructed manifolds, they are not belong to the same dynamical system. In this case one can say that there is no causality between the two variables.
Convergent Cross-Mapping is a procedure which tests the existence or the absence of this mapping. It is cross-mapping, because it estimates the mapping between the two reconstructed manifolds and convergent because this estimate converges to the true mapping as one takes longer and longer time series.



This idea was extended to detect interaction delays between variables (Ye Hao et al., 2015). The more or less parallel work of Schumacher et al. is based on same principles, it also contains time delay detection and in addition they applied their method to neural data (Schumacher et al., 2015).
There were attempts to apply the method to indicate time varying interactions. For example Ma et al used a feed forward neural network to explicitly estimate the smooth mapping between the embedded times series. When the mapping error was small enough they detected causal relation otherwise they said that the two time series were independent in the time segment (Ma et al 2014). Other approaches are also possible to detect time varying Sugihara causality (Somogyvári et al, unpublished).
Sugihara's method works well on deterministic data and in the case of circular causality, but it cannot detect hidden common causes. Moreover the linear mixing, and signal autocorrelation restricts the applicability on raw extracellular or imaging data.



$$F M_x=M \qquad \exists F^{-1}$$
$$G M_y=M \qquad \exists G^{-1}$$
$$G^{-1}F=Q \qquad F^{-1}G=Q^{-1}$$
$$Q M_x=M_y \qquad M_x=Q^{-1}M_y$$