Dynamic Threshold Modeling of Budget Changes

Bryan D. Jones[1], **László Zalányi**[2], **Frank R. Baumgartner**[3] **and Péter Érdi**[2,4]

[1]Department of Government, University of Texas at Austin
[2]Department of Biophysics, KFKI RIPNP of the Hungarian Academy of Sciences, Budapest, Hungary
[3]Department of Political Science, UNC, Chapel Hill, NC
[4]Center for Complex Systems Studies, Kalamazoo College, Kalamazoo, Michigan, USA

Abstract

A family of models was given to explain how the public budgeting process, as a multi-stage institutional decision making mechanism transforms the stimuli characterized by Gaussian distribution to skew, power law distributions. While the annual change is generally incremental, deviations from this incremental changes are more frequent, than the Gaussian distribution suggests. A set of threshold models, reflecting error-accumulation and friction, was suggested. The three-threshold model seems to be good to describe appropriately the basic statistical features of the data.

Introduction

This paper is a first report of a collaborative effort between political scientists and people working on complex systems and related areas.

Two of us (BJ and FB) have published a set of papers, books focusing on annual budget changes (Jones and Baumgartner 2005b). Leptokurtic distribution of percentual budget changes were observed in a broad range of settings: small increases and small decreases of budgets and budget components are the most frequent, but time to time large increases and cut-offs are observed as well. Most frequency distributions appeared as linear on diagrams with double log axes, which strongly suggests a special case of kurtosis, the power-law distribution. Dynamic mechanisms leading to power law are one of the hot topic in the theory and practice of complex systems (Érdi 2007).

We are interested in explaning both of the possible macroscopic processes and motivations of the more microscopic agents involved in the political decision system in a mathematically non-formalized way. Some preliminary **dynamic model** also were set up for simulating the institutional decision processes. The input signals of a decision process are subject of linear and non-linear transformations incorporating error-accumulation, threshold(s) and memory effects. The input signals are considered as Gaussian distributions and we are interested in the institutional mechanisms and the assigned computational algorithms which transforms them to non-Gaussian ones.

Lessons from Political Science

Government budgets set public priorities; they are the outcome of complex policy processes involving the nature of the decision-making institutions, the preferences of decision-makers (organized by political parties), and informational signals from a changing environment. In many real-world information-processing situations, including the study of public budgets, we do not have the luxury of observing the actual informational inputs, because we observe only whether the decision-maker attends to that information and what action he or she subsequently takes. As a consequence, most of the work in the area is in the form of reconstructing decision-making process from the distribution of outputs, supplemented by unfortunately too few direct observational studies.

Early studies of public budgeting emphasized uncertainty in the decision-making environment. Budgeting in the absence of information about the impacts of decisions led to an adjustment process rooted in simple decision rules and bargaining among interests. This led to marginal or incremental adjustments from the budgetary status quo, with all major actors wary of big changes to the budgetary base. Unless, that is, major exogenously generated shifts caused the entire budgeting system to move up or down (Davis, Dempster, and Wildavsky 1966).

There was, however, considerable dissatisfaction with the approach based on quantitative observations of budget outcomes. Sometimes they just didn't look incremental, even adjusting for the regime shifts estimated by (Davis, Dempster, and Wildavsky 1966). Most arguments were regression-based and centered on the size of the increment, which sometimes looked pretty large.

In 1980, John Padget (Padgett 1980) shifted the terms and methodology of the debate from regression studies to the application ot the theory of stochastic processes. He first showed that incremental decision processes implied a Gaussian distribution of first differences, and developed one particular model of decision-making, serial processing. In that model decision-makers faced with a constraint kept moving incrementally in the direction implied by the constraint until they find a point that "satisfices". Hence incremental processes under constraints can lead to non-incremental budget changes - or perhaps more accurately, attempts to be incremental can lead to non-incremental decisions. Padget went on to show that the stochastic process implications of serial decision-making for budget proposals were either an exponential or a power function, depending on whether choices were within a single policy area or a mixture of several.

In the 1990s, Jones, Baumgartner, and True (JBT) initiated a series of investigations based on a new dataset of consistent measures of US Budget Authority since World War II. Budget authority is a better overall measure than outlays, because it is closer to the decision-making process; budget outlays measure when actual expenditures occur. JBT showed that budget distributions were clearly leptokurtic, and definitely non-Gaussian, and that this finding was robust across various time periods and aggregation levels. One example is the US budget change histogram and a fitted Gaussian distribution, see Fig. 1:

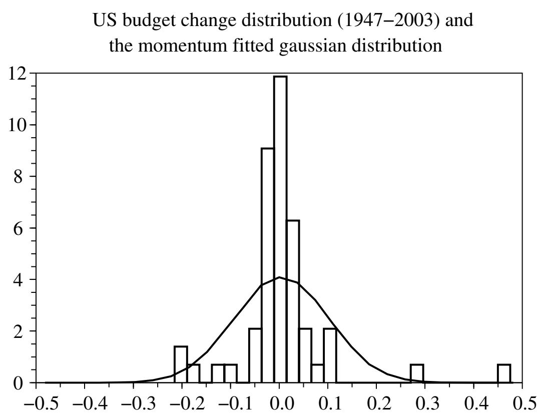

Figure 1: Simple fitting of a Gaussian distribution with the preserved mean and deviation

Data to a first approximation followed a double Paretian or power distribution, because budgets can be cut as well as increased (True, Jones, and Baumgartner 2007). Later investigations by several researchers showed a remarkable similarity in budget distributions across several democracies, but considerable differences in the parameter estimates for the power distributions characterizing the different nations (Jones et al. 2009). The research ruled out the incremental model and was consistent with the serial decision approach.

The next step was to try to develop models of policy change that were broader than either the incrementalist model or Padget's serial processing model, but would be capable of incorporating them under appropriate circumstances. It the case of Padgett's approach, models should get beyond the postulation of constraints to generate the distribution. That is, the operation of the constraints should somehow be incorporated into the model.

The key was the recognition that humans are disproportionate processors of information - that is, they are incapable of adjusting actions directly to the incoming information that has implications for action (Jones 2001). There are a variety of reasons for this inability, but the most important is the inability of people to pay attention to multiple streams of information at the same time. Instead, they juggle among income streams, with emotional arousal being the central adjustment mechanism. This leads to disjoint and episodic shifts in attention to the information streams they face. Punctuations in processing capacity, and hence in any action taken, is a direct result of coping with numerous streams of information in a complex world.

In political systems, there are good reasons to expect that this tendency may be magnified. All systems have some mechanisms for parallel processing of input streams, but when policy action requires policy-making, then the classic Simon "bottleneck of attention" limits the agenda space. Democracies have built-in error correction mechanisms in that if policymakers are not appropriately responding, they may be replaced.

"If we put together the limits of human information processing and the characteristics of democracies that encourage error correction, we get a model of politics that is very static but reluctantly changes when signals are strong enough. The system resists change, so that when change comes it punctuates the system, not infrequently reverberating through it." (Jones and Baumgartner 2005b).

This reluctance was summarized in the twin notions of cognitive and institutional friction. Both the cognitive capacities of actors and the inherent conservative nature of political systems caused a pattern that resembled the stick-slip dynamics observed in the study of earthquakes (Jones, Sulkin, and Larsen 2003; Jones and Baumgartner 2005b) and other "critical phenomena" using the physicists' terminology.

Jones and Baumgartner used the term "error accumulation" to characterize such mechanisms, and developed a single threshold model to begin to account for the strongly leptokurtic budget output distributions they observed. Errors, in the sense of a mis-match between information inputs and policy outputs, would accumulate in a policy area until the errors exceeded a threshold, after which policymakers would attend to the problem and initiate action. Simulations based on the model suggested considerable agreement, but problematic mis-matches in some cases (Jones and Baumgartner 2005a).

Error-accumulation: Single Threshold Models

A dynamic model with an error-accumulating threshold-based non-linearity and an infinite memory of earlier signals. there is a single threshold, if the accumulated input is below the threshold, the response is small, if it is above above than large.

The variables and parameters of the model can be summarized as follows:

R_t: response variable
S_t: input signal, (Gaussian random variable)
C: threshold
λ : efficiency parameter or friction
β: amplification parameter.

Error accumulation (incorporated as sum of the signals) is a key feature of the model. $0 < \lambda < 1$, the efficiency below the threshold C is lower the one, but definitely shold be lower than β. This is one of the characteristic point of the model: below the threshold there are smaller reaction than it is above the threshold. The model has two versions. Error accumulation can or cannot be reset to zero,

$$R_t = \begin{cases} \beta S_t & \text{if } \sum_{t'=k}^{t-1} S_{t'} > C, \\ \lambda\beta S_t & \text{otherwise} \end{cases}$$

where k is the time of the resetting ($k = 1$ means no reset).

Simulations results for two situations (without and with resetting) are shown in Fig. 2. Appropriate choice of the λ, β, C parameters makes this model reach the actual kurtosis values observed on real data. $C = 1$, $\lambda = 0.4$ and $\beta = 1$ were chosen.

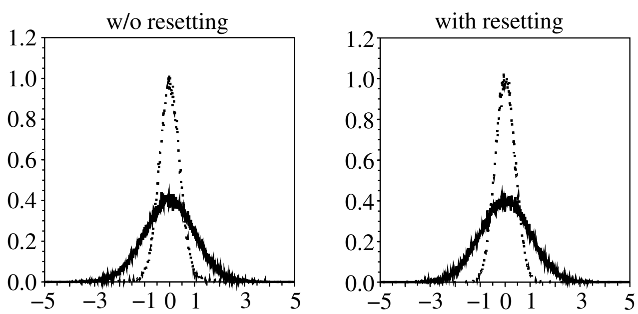

Figure 2: A leptokurtic distribution as a simulated response, comparing to a Gaussian distribution

.

What is the difference between the two models? The counter spends a lot of time above the threshold and also below in the first case. The higher the threshold the narrower the output distribution (increasing the kurtosis) but the rarer the "overshooting", i.e. the big changes. The resulted output distribution is a sum of two Gaussians. In the second case the output much narrower as we restrict the "time" what the counter can spend above the threshold, and hence the weight of the wider Gaussian (β) in the sum is smaller. However, there is one more difference. It is hardly seen that the output distributions are asymmetric (and hence they are not exactly sums of Gaussians, but can be approximated very well). This asymmetry comes from the requirements of threshold crossing direction: it can only be crossed in the positive direction to switch into the frictionless state.

In summary, the single threshold model cannot explain the higher frequency of the occurrence of some extreme events.

Multiple Threshold Models

Two-threshold Models

We might improve the fit to real date by introducing some more thresholds, and for the different intervals different frictions. First, a low threshold was introduced to take into account explicitly the dying out of some topics. On Fig. 3 the effect of this lower threshold is shown at three different threshold values. When the lower threshold is "strict", i.e. it is 0, the counter is not allowed the accumulate negative values, so all random input signals which exceed the upper threshold (here it is $C2 = 1$) remains unchanged both in frequency and "size". This is shown on the upper figure, the positive tail follows exactly the input tail. Allowing some accumulation of negative values reduces the sharp hop at the upper threshold. The output distribution is asymmetric, although the negative side is still a compressed ($\lambda = 0.4$) Gaussian.

The next extension is to describe some adaptation by making the upper threshold a little noisy, see Fig. 4. The noisy threshold is modeled by an additive white noise with zero mean and 0.2 standard deviation. The noisy threshold softens the sharp changes seen in Fig. 3.

The two-threshold model reflects the important features of the budget change distributions. However, budget changes distributions are not only narrow and asymmetric with fat tails but they have a shoulder around the central peak, that is the distribution does not decrease to zero so sharply as a compressed Gaussian does. The introduction of one more threshold helps.

Three-threshold Models

The institutional decision making is obviously a multi-stage process. The simplest way to incorporate the series of processes into a model is to increase the number of thresholds. Different frictions for negative and positive counter (i.e. demand) were defined. Also it was necessary to smooth the positive side of the output distribution which can be remedied by introducing a second, upper threshold for the real overshooting reactions and a specific friction for the intermediate regime. This model gives different responses in the negative counter depending on the sign of the actual input: the response is different to an increase then to a decrease demand. By incorporating these modifications a much finer fit to the budget changes distributions was obtained.

$C_1 = -4$; lower threshold
$C_2 = 1.2$; middle threshold
$C_3 = 3$; upper threshold
$\lambda_{1n} = 0.02$; efficiency for the region of negative accumulated sum
$\lambda_{1p} = 0.03$; efficiency for the region of positive accumulated sum
$\lambda_{2p} = 0.3$; efficiency between the two positive threshold
$\beta = 1$; and

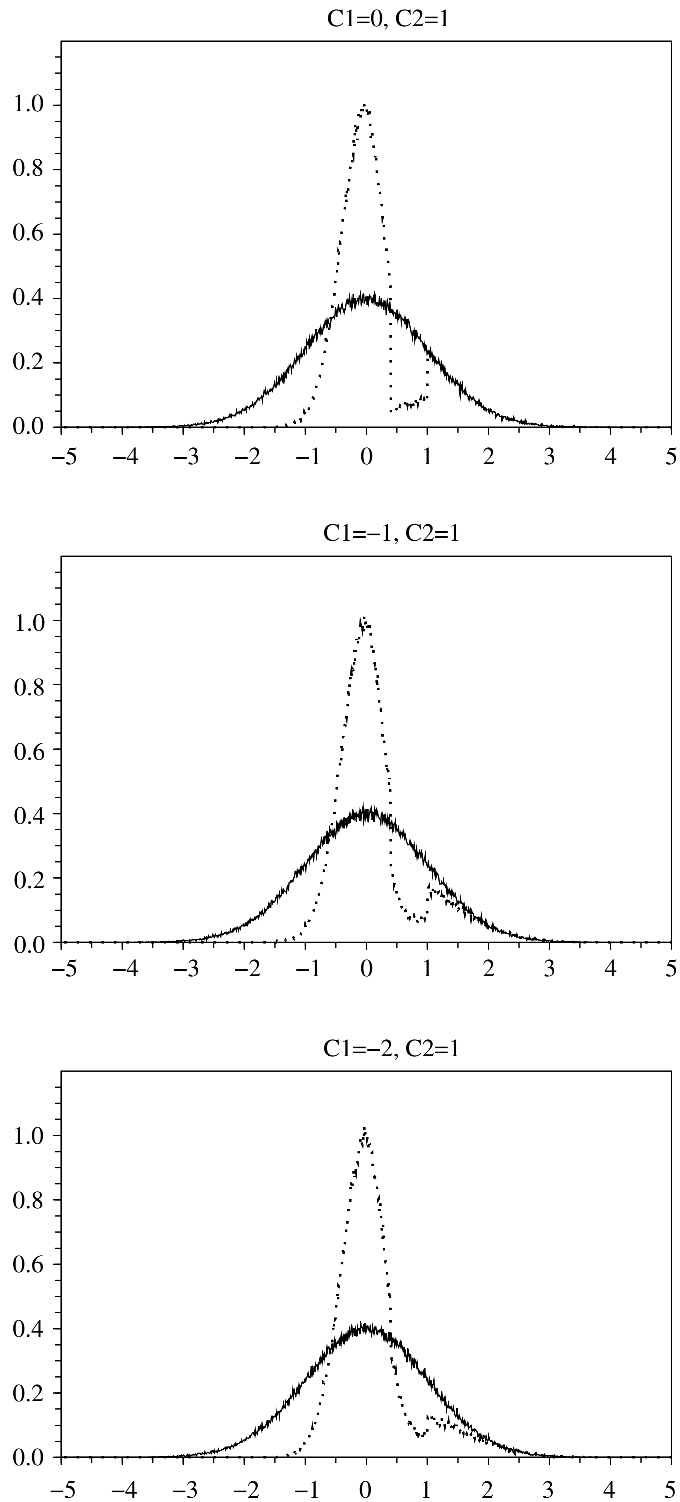

Figure 3: Distribution with a low threshold with three different values compared to a Gaussian distribution

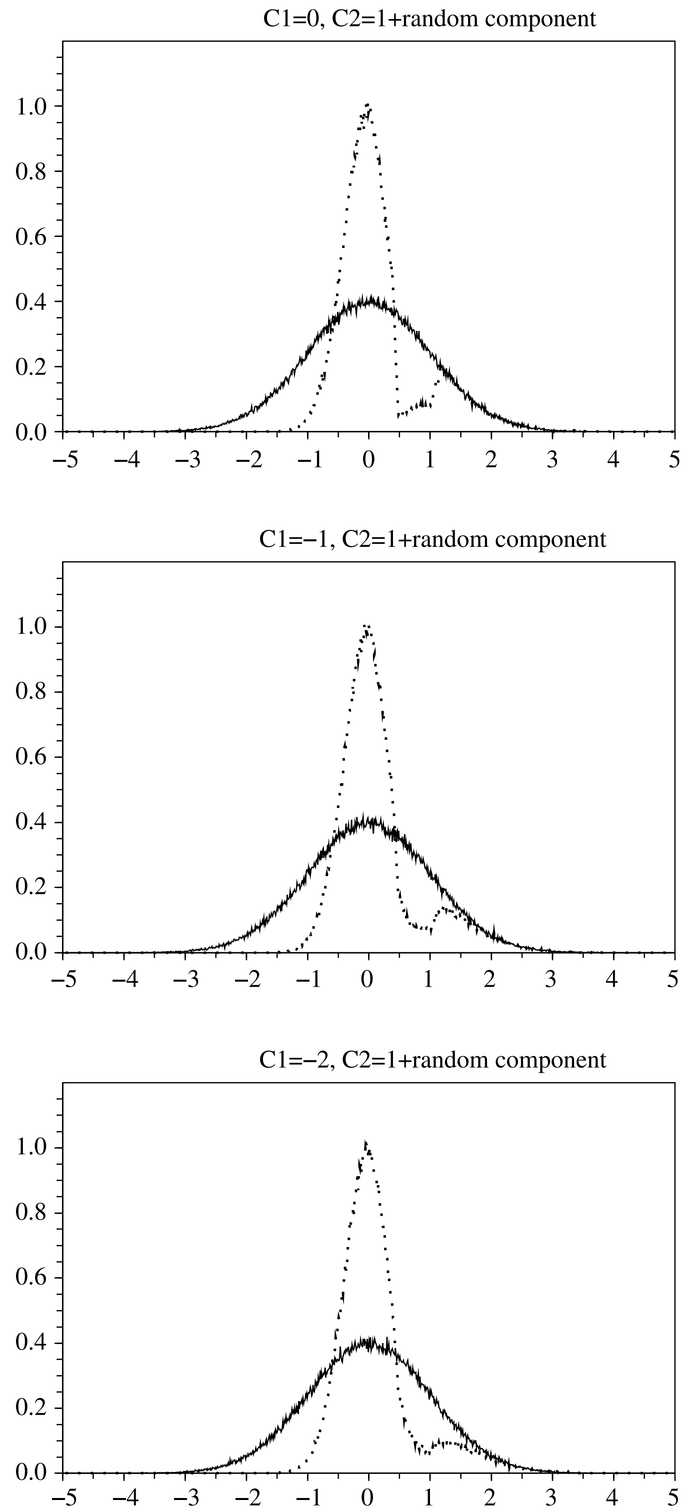

Figure 4: Distribution with three different low thresholds and a noisy upper threshold compared to a Gaussian distribution

T denotes the time of last resetting to 0 of accumulated signal.

The response

$$R_t = \begin{cases} \beta S_t & \text{if } \sum_{t'=T}^{t-1} S_{t'} > C_3, T = t \\ \lambda_{2p}\beta S_t & \text{if } C_3 > \sum_{t'=T}^{t-1} S_{t'} > C_2 \\ \lambda_{1n}\beta S_t & \text{if } \sum_{t'=T}^{t-1} S_{t'} < 0 \wedge S_t < 0 \\ \lambda_{1n}\beta S_t & \text{if } \sum_{t'=T}^{t-1} S_{t'} < C_1, T = t \\ \lambda_{1p}\beta S_t & \text{if } \begin{array}{l} C_2 > \sum_{t'=T}^{t-1} S_{t'} > 0 \vee \\ 0 > \sum_{t'=T}^{t-1} S_{t'} > C_1 \wedge S_t > 0. \end{array} \end{cases}$$

Simulation results

Here just two specific results are shown. Fig. 5 shows the simulation of the French budget data with the fitted parameters:

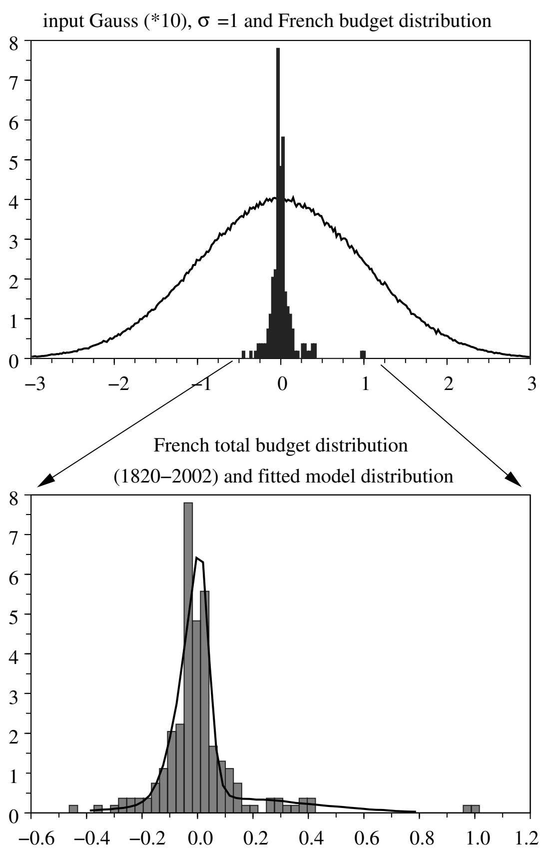

Figure 5: Transformation of the Gaussian input signals to skew distribution fitted to the French budget data. The upper figure shows the tenfold enlarged input Gaussian sample distribution and the budget distribution histogram. The lower figure shows the measured data histogram and the fitted model output histogram (solid line). Parameters of the fit: $C_1 = -3$; $C_2 = 1.6$; $C_3 = 4.6$; $\lambda_{1n} = 0.07$; $\lambda_{1p} = 0.04$; $\lambda_{2p} = 0.3$.

Fig. 6 shows the US budget data with the fitted model distribution. The fittings were performed as follows: numerically explored the six-dimensional parameter space and the set of parameters which gives the maximum likelihood of the data-set was selected. The three-threshold model fits are better then that of the former models. It reproduces generally the shape, the asymmetric feature of the distribution of the data, and gives significant probability of extreme events. However, the model is not perfect: both the mean and standard deviations of the simulated distributions are bigger then the momenta of data-sets. The reason of this deviation partially comes from the features of the model: it is a continuous one which has to fit to a rather sparse data-set. There are several ways to extend or modify the threshold models. To make the next step one has to investigate the decision-making procedure of the different countries in details due to the natural limitation of the annual budget data sizes.

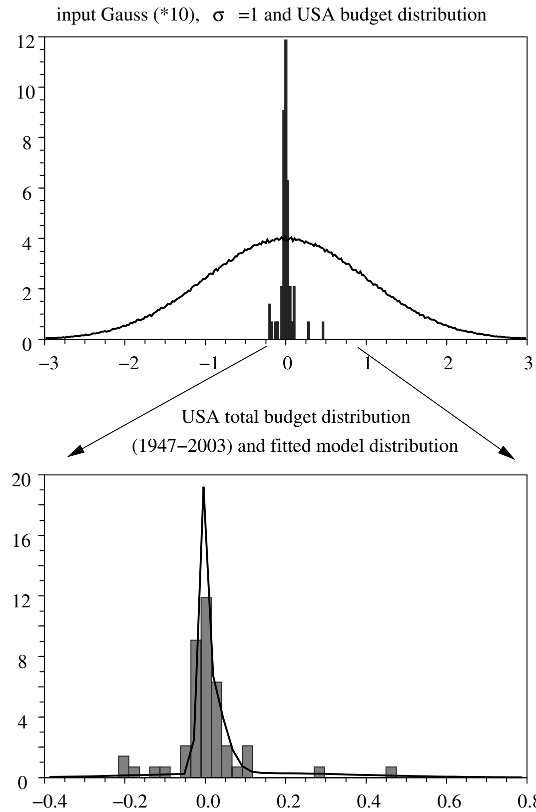

Figure 6: Transformation of the Gaussian input signals to skew distribution fitted to the US budget data. The upper figure shows the tenfold enlarged input Gaussian sample distribution and the budget distribution histogram. The lower figure shows the measured data histogram and the fitted model output histogram (solid line). Parameters of the fit: $C_1 = -12$; $C_2 = 0$; $C_3 = 3$; $\lambda_{1n} = 0.01$; $\lambda_{1p} = 0.04$; $\lambda_{2p} = 0.3$.

Discussion

Our general goal in this cooperation is to integrate the the conceptual-empirical and computational perspectives of the political decision making processes. The existing models underlying the complex organizational decision-making for government policy proved to be not satisfactory. Systems characterized by friction remain ”stable” until the signals from outside exceed a threshold. Our analysis suggests that it is more likely there are multiple thresholds in the process. What we hope to show in the future that institutional decision making mechanisms can be better understood by using the concepts and models of the theory of complex systems.

Preliminary cooperation with Gábor Borgulya in formulating models, and with János Minich and János Tóth about statistical problems are acknowledged.

References

Davis, O.; Dempster, M.; and Wildavsky, A. 1966. A theory of the budget process. *American Political Science Review* 60:529–547.

Érdi, P. 2007. *Complexity Explained*. Springer Verlag.

Jones, B. D., and Baumgartner, F. R. 2005a. A model of choice for public policy. *Journal of Public Administration Theory* 15:325–351.

Jones, B. D., and Baumgartner, F. R. 2005b. *The Politics of Attention: How Government Prioritizes Problems*. University of Chicago Press.

Jones, B. D.; Baumgartner, F. R.; Breunig, C.; Wlezien, C.; Soroka, S.; Foucaul, M.; Francois, A.; Green-Pedersen, C.; Koske, C John, P.; Moretensen, P. B.; Varone, F.; and Walgrav, S. 2009. A general empirical law of public budgets. *American Journal of Political Science* 53:855–873.

Jones, B. D.; Sulkin, T.; and Larsen, H. 2003. Policy punctuations in American institutions. *American Political Science Review* 97:151–170.

Jones, B. D. 2001. *Politics and the Architecture of Choice*. University of Chicago Press.

Padgett, J. F. 1980. Bounded rationality in budgetary research. *American Political Science Review* 74:354–72.

True, J.; Jones, B. D.; and Baumgartner, F. R. 2007. Punctuated equilibrium theory: Explaining stability and change in policymaking. In Sabbatier, P. A., ed., *Theories of the Policy Process*. Boulder, CO: Westview Press.